BIRINCHI TARTIBLI CHIZIQLI DIFFERENSIAL TENGLAMALAR FIZIK MASALALARDA

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Annotatsiya: *Ushbu maqolada matematika fanining asosiy bo'limlaridan biri differensial tenglamalar haqida ma'lumot berilgan. Asosan maqolada birinchi tartibli chiziqli differensial tenglamalar, o'rnigaqo'yish (Bernulli usuli), o'zgarmasni variatsiyalash (Lagranj usuli) va ularni fizika masalarda tadbiqi ko'rsatilgan.*

Kalitso'zlar: *Differensial tenglama, birinchi tartibli differensial tenglama, birinchi tartibli chiziqli differensial tenglama, Bernulli usuli, Lagranj usuli.*

ПЕРВОГО ПОРЯДКА ЛИНЕЙНЫЕ ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ В ФИЗИЧЕСКИХ ЗАДАЧАХ

Аннотация: *В данной статье представлена информация о дифференциальных уравнениях, которые являются одном из основных частей математических наук. В основном в статье рассматриваются дифференциальные уравнения первого порядка, метод подстановки (метод Бернулли), вариация постоянной (метод Лагранжа) и их применение в физических задачах.*

Ключевые слова: *Дифференциальное уравнение, дифференциальное уравнение первого порядка, линейные дифференциальные уравнения первого порядка, метод Бернулли и метод Лагранжа.*

FIRST-ORDER LINEAR DIFFERENTIAL EQUATIONS IN PHYSICAL PROBLEMS

Annotation*:This article provides information about differential equations, which is one of the main branches of mathematics. The article mainly discusses first-order linear differential equations, substitution (Bernoulli method), variation of parameters (Lagrange method) and their application in physical problems.*

Keywords: *Differential equation, first-order differential equation, first-order linear differential equation, Bernoulli method and Lagrange method.*

Atrofimizda sodir bo'layotgan ko'pgina hodisalar va jarayonlar, ularni tavsiflaydigan noma'lum funksiyalar yoki ularning hosilasi qatnashgan tenglamalar orqali ifodalanadi. Biz fizika, iqtisodiyot, amaliyotda va boshqa sohalarda differensial tenglamadan foydalanamiz.

Erkli o'zgaruvchi x, noma'lum funksiya y=f(x) hamda uning hosilalari $y', y'', ..., y^{(n)}$ orasidagi bog'lanishni ifodalovchi tenglama differensial tenglama deyiladi. Sodda aytganda: "Noma'lum funksiya va uning hosilasi qatnashgan tenglikka differensial tenglama deyiladi"[1]. Bu tenglamalardan noma'lum funksiyani toppish kerak. Masalan, biz fizikada ko'radigan bo'lsak:

1-masala. Massasi m bo'lgan jism biror balandlikdan yerga tashlab yuborilgan. Bu jismning tushish tezligi ϑ qanday qonun bilan o'zgaradi? ϑ =ϑ (t) munosabatni topish talab qilinadi.

Yechish. Nyutonning ikkinchi qonuniga ko'ra,

F=ma; (1)

bu yerda: m – jism massasi; a – jism tezlanishi; F – ta'sir etuvchi kuch. Bunda ikki hol bo'lishi mumkin:

a) jismga havoning qarshiligi hisobga olinmagan hol;

Jismga havoning qarshiligi olinmasa, jism faqat og'irlik kuchi ta'sirida harakatlanadi, ya'ni F=mg ga teng bo'ladi. U holda (1) dan quyidagi tenglamaga ega bo'lamiz:

$m\frac{dv}{dt} = mg$ yoki $\frac{dv}{dt} = g;$ (2)

(2) – tezlikka nisbatan birinchi tartibli differensial tenglama.

b) jismga havoning tezligiga proportsional bo'lgan qarshilik kuchi ta'sir etgan hol;

Bunda $F_{qarshilik}$=ρυ bo'ladi. Bu yerda: ρ – proportsionallik koeffitsiyenti, ϑ – jismning tezligi. Bu holda F=mg-$F_{qarshilik}$ kuch ta'sir etadi. U holda (1) dan:

$m\frac{dv}{dt} = mg - \rho\vartheta \quad yoki \quad \frac{dv}{dt} = g - \frac{\rho}{m}\vartheta$ (3)

Biz yana birinchi tartibli differensial tenglamaga ega bo'lamiz.

Noma'lum funksiya va uning hosilasiga nisbatan chiziqli bo'lgan (1-darajali) differensial tenglamaga chiziqli differensial tenglama deyiladi.

$\frac{dy}{dx} + P(x)y = Q(x)$ (4)

Bunda: P(x) va Q(x) lar x ning berilgan uzluksiz funksiyalari (yoki o'zgarmas sonlar). Agar Q(x)=0 bo'lsa, (4) tenglama bir jinsli, aks holda bir jinsli bo'lmagan chiziqli differensial tenglama deyiladi. Bir jinsli bo'lmagan chiziqli differensial tenglamani yechishni 2 ta usuli:

1.O'rniga qo'yish (Bernulli usuli);

2.O'zgarmasni variatsiyalash usuli (Lagranj usuli).

(4) ni o'zgarmasni variatsiyalash yoki Lagranj usulida ishlaymiz. Q(x)=0 shart kiritamiz. O'zgarmasni variatsiyalash usulida birinchi navbatda berilgan tenglamaning bir jinsli qismini ishlab olamiz:

$$y' + P(x)y = Q(x)$$

$$y' + P(x)y = 0 \;\leftrightarrow\; y' = -P(x)y$$

$$\frac{dy}{dx} = -P(x)y \;\leftrightarrow\; \int\frac{dy}{y} = -\int P(x)dx$$

$$ln|y| = -\int P(x)dx + lnC \;\leftrightarrow\; y = Ce^{-\int P(x)dx}$$

Berilgan tenglamaning umumiy yechimini y= $Ce^{-\int P(x)dx}$ ko'rinishida qidiramiz:

$$y' = C'e^{-\int P(x)dx} + C(-P(x))e^{-\int -P(x)dx}$$

$$C'e^{-\int P(x)dx} + C\big(-P(x)\big)e^{-\int -P(x)dx} + P(x)Ce^{-\int P(x)dx} = Q(x)$$

$$C'e^{-\int P(x)dx} = Q(x) \leftrightarrow C(x) = \int Q(x)e^{\int P(x)dx}dx + C_1$$

y=$e^{-\int P(x)dx}(\int Q(x)e^{\int P(x)dx}dx + C_1)$;

O'rniga qo'yish usulida noma'lum funksiya y=u(x)v(x) ko'rinishida qidiramiz.

y'+P(x)y=Q(x) shu misolni o'rniga qo'yish usulida ishlaymiz.

$$y = u(x) \cdot v(x) \leftrightarrow y' = u' \cdot v + u \cdot v'$$

$$u' \cdot v + u \cdot v' + P(x) \cdot u \cdot v = Q(x) \leftrightarrow u' \cdot v + u \cdot [v' + P(x) \cdot v] = Q(x)$$

Bernulli usulini mohiyati shundaki, qavs ichidagi ifodani nolga teng deb qarashdir. Biz ham qavsichidagi ifodani nolga tenglaymiz:

$$v' + P(x) \cdot v = 0 \leftrightarrow v' = -P(x) \cdot v \leftrightarrow \frac{dv}{dx} = -P(x) \cdot v$$

$$\int \frac{dv}{v} = -\int P(x)dx \leftrightarrow ln|v| = -\int P(x)dx$$

$$v(x) = C \cdot e^{-\int P(x)dx} \quad \leftrightarrow \quad v(x) = e^{-\int P(x)dx}$$

Bunda biz C=1 deb olishimiz mumkin. Chunki, hozir biz υ(x) nitopganimizdagi C – o'zgarmas son muhim emas. Lekin u(x) funksiyani topgandagi C – o'zgarmas son muhim.

$$u' \cdot v + u \cdot 0 = Q(x) \leftrightarrow u' \cdot e^{-\int P(x)dx} = Q(x) \leftrightarrow u' = Q(x) \cdot e^{\int P(x)dx}$$

$$\frac{du}{dx} = Q(x) \cdot e^{\int P(x)dx} \leftrightarrow \int du = \int Q(x) \cdot e^{\int P(x)dx}dx$$

$$u(x) = \int Q(x) \cdot e^{\int P(x)dx}dx + C$$

$$y = u \cdot v = v = e^{-\int P(x)dx}\Big(\int Q(x) \cdot e^{\int P(x)dx}dx + C\Big)$$

2-masala.Massasi m,issiqlik sig'imi C o'zgarmas bo'lgan jism boshlang'ich momentda T_0 temperaturaga ega bo'lsin. Havo temperaturasi o'zgarmas va τ (T>τ) ga teng. Jismning cheksiz kichik dt vaqt ichida bergan issiqligi jism va havo temperaturalari orasidagi farqqa, shuningdek vaqtga proporsional ekanligini e'tiborga olgan holda jismning sovish qonunini toping.

Sovuyotganda jism temperaturasi T_0 dan τ gacha pasayadi. Vaqtning boshlang'ich t vaqtda jism temeraturasi T bo'lsa, cheksiz kichik dt vaqt oralig'ida jism bergan issiqlik miqdori:

$$dQ = -C \cdot (T - \tau)dt$$

ga teng. Bu yerda, C – issiqlik sig'imi.

Jism T temperaturadan τ temperaturagacha soviganda beradigan issiqlik miqdori:

$$dQ = c \cdot m \cdot (T - \tau)dt \leftrightarrow dQ = c \cdot m \cdot dT$$

Ikkala ifodani tenglaymiz:

$$-C \cdot (T - \tau)dt = c \cdot m \cdot dT \leftrightarrow \int \frac{dT}{T - \tau} = -\int \frac{C}{c \cdot m}dt$$

$$ln|T - \tau| = -\frac{C}{c \cdot m}t + lnC_1 \leftrightarrow T - \tau = C_1 \cdot e^{-\frac{C}{c \cdot m}t}$$

Boshlang'ich shart beradigan bo'lsak, C_1 ni topish qulayroq. t=0 da T=T_0 bo'lsin, bunda C_1=T_0-τ ga teng.

$$T-\tau=(T_0-\tau)\cdot e^{-\frac{C}{c\cdot m}t} \quad \leftrightarrow \quad T=\tau+(T_0-\tau)\cdot e^{-\frac{C}{c\cdot m}t}$$

3-masala. m massali nuqta vaqtga proporsional bo'lgan kuch ta'sirida to'g'ri chiziqli harakat qilmoqda. Boshlang'ich t=0 vaqt momentida v=0 bo'lsin. Havo qarshiligitezlikkaproporsionalbo'lganholdatezlikni t ningfunksiyasisifatidaaniqlang.

Bunda t momentdanuqtagaikkitakuch, ya'ni vaqtga proporsional bo'lgan kuch va havoning qarshilik kuchi ta'sir qiladi:

$$F=F_1+F_2=g\cdot t+(-k\cdot v)=g\cdot t-k\cdot v$$

Nyutonning ikkinchi qonuniga ko'ra, $F=m\cdot a=m\frac{dv}{dt}$ ga teng:

$$m\frac{dv}{dt}=g\cdot t-k\cdot v \quad \leftrightarrow \quad m\frac{dv}{dt}+k\cdot v=g\cdot t$$

Tenglikni har ikkala qismini m massaga bo'lib yuboramiz:

$$\frac{dv}{dt}+\frac{k}{m}v=\frac{g}{m}t \quad \leftrightarrow \quad v'+\frac{k}{m}v=\frac{g}{m}t$$

Bu masalani yuqorida ko'rib o'tganimizdek, Bernulli usuilida ishlaymiz:

$$v=u(t)\cdot w(t) \quad \leftrightarrow \quad v'=u'\cdot w+u\cdot w'$$

$$u'\cdot w+u\cdot w'+\frac{k}{m}u\cdot w=\frac{g}{m}t \quad \leftrightarrow u'\cdot w+u\cdot\left[w'+\frac{k}{m}w\right]=\frac{g}{m}t$$

$$w'+\frac{k}{m}w=0 \quad \leftrightarrow \quad w'=-\frac{k}{m}w$$

$$\frac{dw}{dt}=-\frac{k}{m}w \quad \leftrightarrow \quad \int\frac{dw}{w}=-\int\frac{k}{m}dt$$

$$ln|w|=-\frac{k}{m}t+lnC \quad \leftrightarrow \ll C=1\gg \leftrightarrow \quad w=e^{-\frac{k}{m}t}$$

$$u'\cdot w+u\cdot 0=\frac{g}{m}t \quad \leftrightarrow \quad u'\cdot e^{-\frac{k}{m}t}=\frac{g}{m}t$$

$$u'=\frac{g}{m}\cdot e^{\frac{k}{m}t}\cdot t \quad \leftrightarrow \quad \frac{du}{dt}=\frac{g}{m}\cdot e^{\frac{k}{m}t}\cdot t$$

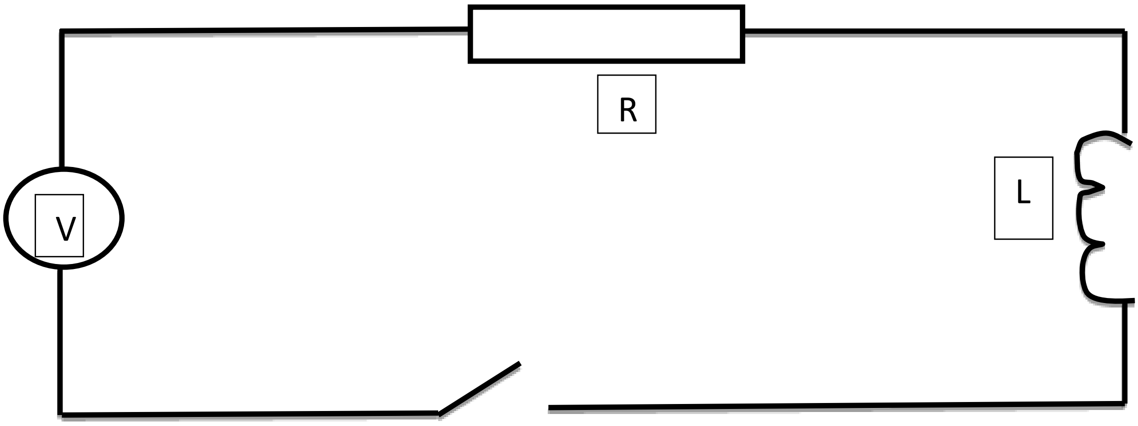
$$\int du=\int\frac{g}{m}\cdot e^{\frac{k}{m}t}\cdot tdt \quad \leftrightarrow \quad u=\frac{g}{m}\cdot\left(t-\frac{m}{k}\right)\cdot e^{\frac{k}{m}t}+C$$

$$v=u(t)\cdot w(t)=e^{-\frac{k}{m}t}\cdot\left(\frac{g}{m}\cdot\left(t-\frac{m}{k}\right)\cdot e^{\frac{k}{m}t}+C\right)$$

4-masala.Tok manbaiga E=Vsinωt qonun bo'yicha o'zgaruvchi kuchlanish, R qarshilik va L induksiya ketma-ket ulangan. Zanjirdagi tok kuchini toping (barqaror holat).

Krigroffning ikkinchi qonunini yozamiz, unga ko'ra oqim manbalarining kuchlanishlari yig'indisiga teng:

$$I\cdot R+L\frac{dI}{dt}=Vsin\omega t$$



Biz barqaror tok holatni topishimiz kerakligi sababli:

$I = a \cdot sin(\omega t - \varphi)$ (5)

Bu yerda: a – amplituda, φ – faza. Biz endi φ ni topamiz:

$$R \cdot a \cdot sin(\omega t - \varphi) + L \cdot a \cdot \omega \cdot cos(\omega t - \varphi) = V \cdot sin\omega t$$

$$R \cdot a(sin\omega t \cdot cos\varphi - cos\omega t \cdot sin\varphi) + L \cdot a \cdot \omega(cos\omega t \cdot cos\varphi + sin\omega t \cdot sin\varphi) = V \cdot sin\omega t$$

$$sin\omega t(R \cdot a \cdot cos\varphi + \omega \cdot L \cdot a \cdot sin\varphi) + cos\omega t(L \cdot a \cdot cos\varphi - R \cdot a \cdot \omega \cdot sin\varphi) = V \cdot sin\omega t$$

Sinus va kosinuslarning o'ng va chap tomonlarini tenglashtiramiz:

$$\begin{cases} a(R \cdot cos\varphi + \omega \cdot L \cdot sin\varphi) = V \\ a(\omega \cdot L \cdot cos\varphi - R \cdot sin\varphi) = 0 \end{cases}$$

Ikkinchi sistemadan:

$$R \cdot sin\varphi = L \cdot \omega \cdot cos\varphi \quad \leftrightarrow \quad tg\varphi = \frac{\omega \cdot L}{R}$$

Biz sinus bilan kosinusni tangens bilan ifodaladik. Bunda trigonometrik almashtirishlar qildik:

$$1 + (tg\varphi)^2 = \frac{1}{(cos\varphi)^2}$$

$$\frac{1}{(cos\varphi)^2} = 1 + \left(\frac{\omega \cdot L}{R}\right)^2 \quad \leftrightarrow \quad (cos\varphi)^2 = \frac{R^2}{R^2 + \omega^2 \cdot L^2}$$

$$(sin\varphi)^2 = 1 - (cos\varphi)^2 = \frac{\omega^2 \cdot L^2}{R^2 + \omega^2 \cdot L^2}$$

Topgan ifodalarimizni sistemaning birinchi tenglamasiga qo'yamiz:

$$a\left(\frac{R^2 + \omega^2 \cdot L^2}{\sqrt{R^2 + \omega^2 \cdot L^2}}\right) = V \quad \leftrightarrow \quad a \cdot \sqrt{R^2 + \omega^2 \cdot L^2} = V$$

Shunday qilib, tebranish amplitudasini topamiz:

$$a = \frac{V}{\sqrt{R^2 + \omega^2 \cdot L^2}}$$

Amplitudasini topganimizdan so'ng, φ fazani topamiz:

$$\varphi = arctg\frac{\omega \cdot L}{R}$$

Topgan ifodalarimizni (5) ga qo'yib, tok kuchini topamiz:

$$I = a \cdot sin(\omega t - \varphi) \quad \leftrightarrow \quad I = \frac{V}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot sin(\omega t - arctg\frac{\omega \cdot L}{R})$$

Xulosa.Yuqoridagi yechilgan masalalardan ko'rinib turibdiki, biz differensial tenglamalarni qayerda qo'llanilishini bilib oldik. Fizika masalalarida differensial tenglamalarni tadbiqini ko'rib chiqdik. Differensial tenglamalarni fizikani mexanika, molekulyar fizika va termodinamika, elektrostatika bo'limlariga oid masalalarida qanday foydalanganligini masalalar orqali tekshirdik.

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