SYSTEMATIC ANALYSIS OF DETERMINING THE SUSTAINABILITY OF TECHNOLOGICAL PROCESSES USING STATISTICAL METHODS

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Annotation. *This article analyzes issues of stability and its provision as an important factor in product quality control during technological process operations. It describes the fact that stability is a part of statistical quality assurance, the method of Shewhart's control charts, and several existing practical methods of stability. The authors have found new control charts for determining the stability of technological processes modeled by a one-dimensional or two-dimensional normal distribution. With their help, it will be possible to improve product quality and manage quality.*

Keywords: *stability, security chart, statistical quality assurance, one-dimensional or two-dimensional normal distribution*

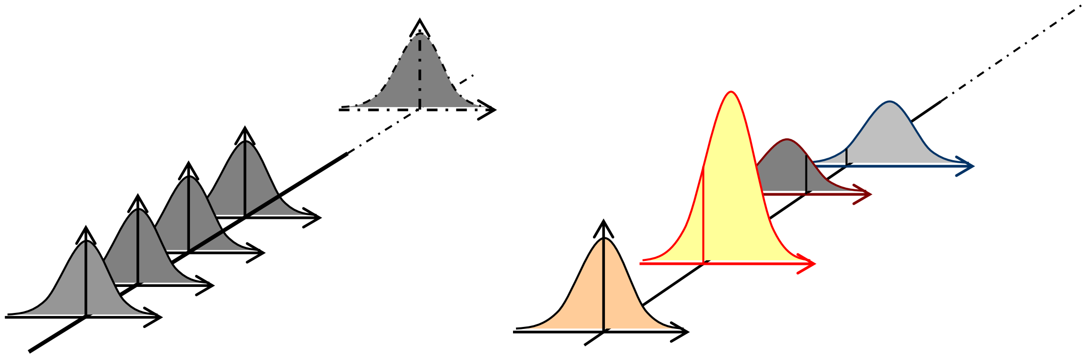
Today, increasing the quality of products and reducing the number of defective products by ensuring the stability of the technological process (TP) is a pressing issue. Determining the stability of TP plays a crucial role in this. Currently, the development of numerical algorithms and software tools for improving the model of control charts (CCs) for studying the stability of TPs is one of the pressing issues. Scientific and technical personnel from industrially developed countries around the world, including the United States, Japan, China, Germany, Korea, Great Britain, the Russian Federation, and other countries, are engaged in such issues as pressing issues.

Scientific research on solving the problems of product quality management using statistical methods was conducted abroad by American scientists and practitioners W. Shewhart, E. Deming, D. Juran, and others, who achieved high results using the CC method in statistical quality assurance (SQA). The theoretical and practical works of scientists such as A.N. Kolmogorov, Yu.K. Belyaev, B.V. Gnedenko, L.N. Bolshev, N.V. Smirnov, D. Khimmeloblau, E. Shindovsky, O. Schurts, R. Storm, B. Hensen, and G. Tagutu also made a significant contribution to the development of this field. Currently, articles and books by scholars such as B. Woodall, J.P. Adler, D. Montgomery, H.Y. Mittag, H. Rinne, V.A. Spear, S.F. Zhulinsky, M. Aslam, B. Costa, R. Kuinino, D. Wheeler, and D. Chambers are of great interest in the development of the CC method and the increase in scientific and methodological work in this field. Therefore, the number of scientific works on the study of processes using CC is increasing. Analysis of research in this field has shown that in the production process, an attempt was made to create a stable TP using models that ensure the stability of the TP using Shewhart's TPs. This method began to be used in industrial enterprises, medicine, education, public administration, small and medium-sized businesses, and other fields. Mathematical models based on CCs that ensure stability have not been found. W. Shewhart created a statistical tool called CC, which reduce variation and costs.

The global scientific and technical community did not pay attention to the CCs statistical tool created by Shewhart for 50 years. Shewhart's followers, E. Deming and G.F. Dodge, saw the potential possibilities of this CCs method. Especially E. Deming was able to demonstrate the possibilities of this method to the «Council of Engineers and Scientists of Japan» and the CCs method played an important role in the restoration of the Japanese economy. CCs has become one of the seven statistical tools used in the SQA by the Japanese. After that, this method began to be used in developed enterprises in America and Europe. It has been used in Russia since the 1990s, and in our republic since the arrival of joint ventures.

It is known that the stability of TP is understood as the property of TP, which determines the reproducibility of the probability distribution according to its parameters over a specified period of time without external intervention. The stability of TP is understood as the ability to ensure the accuracy of product quality parameters over time.

Figure 1 shows the probability distribution of a) stable TP; b) unstable TP as a density function.



a) b)

Figure 1. a) stable TP; b) unstable TP.

The quality indicators of the products manufactured during technological operations in the production facilities depend on the stability of the production facilities. Figure 1 shows that in the a) case, the proportion of high-quality products is high and the process can be predicted, while in the b) case, the proportion of high-quality products is low and the process cannot be predicted.

a) a model of Shewhart's CC (ShCC). Shewhart created rules for determining the stability of the process based on the location of points relative to the LCL and UCL lines, called the lower and upper boundaries of the CCs, and the CL line, called the middle line [14, p.126-127] Based on this, the interval 6σ is divided into 3 upper and 3 lower equal parts relative to the middle line. And the following 4 «Western Electric Territorial Traits» have been developed to determine the violation of stability:

Rule 1: 1 point falls out of the 3σ interval;

Rule 2: 3 consecutive points located on the same side of the midline, at least 2 of which fall outside the interval 2σ.

Rule 3: At least 4 out of 5 consecutive points located on the same side of the midline fall outside the interval 1σ.

Rule 4: At least 8 points on one side of the midline.

If any of this occurs, TP will not be stable. This method is called the ShCC model for determining stability.

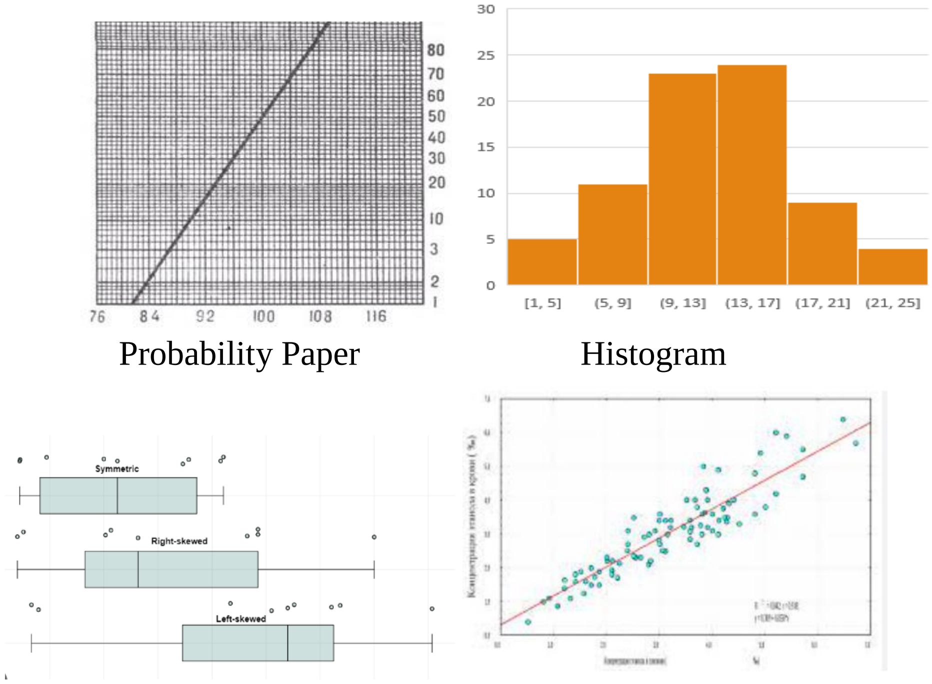
By this method, it is possible to analyze processes even when the distribution of the random variable g is unknown and arbitrary.

b) Other methods for checking the stability of the TP.

When working with ShCCs, it is important to know the stability of TP. In this case, there are statistical tools that are used in conjunction with CCs.

The stability of the process is assessed using tools such as «Histogram», «Data histogram», «Histogram of mean values», «Histogram of range of variation» depending on whether they are within the standard boundaries or within the limits of natural scattering (a-3σ, a+3σ).

In practice, the statistical tools «Histogram», «Box Diogram», «Probability Paper» and «Correlation Coefficient» are used together to determine the stability of TP. If 3 of these 4 methods confirm the normality of the process, then TP is considered stable [5, p. 85-95, p. 169-171; 13, p. 15-25]. Figure 2 below schematically shows these four methods for checking the data obtained from TP for normality:



Probability Paper Histogram

Box Diogram Correlation Coefficient

Figure 2. Methods for checking data for normality.

Analysis of the literature on the study of TP stability shows that stability is primarily analyzed based on the parameters of the distribution of TP characteristics. The correspondence of the empirical distribution of instant samples, determined in technological operations, to the nominal normal distribution, is checked using compatibility criteria.

In determining our results, we will use compatibility criteria based on the sample coefficients of asymmetry and accuracy, as well as Kolmogorov-Smirnov type criteria [6, p.99-100; 7, p.259-260; 9, p.259-269]. The components of the two-dimensional numerical feature are not linearly dependent, and the dependence is carried out using the Student's t-test and Fisher's t-test [1, p.131-134; 2, p.463-467; 8, p.104-108; 15, p.145-200]. Based on these criteria, mathematical models have been developed that determine the stability of the technological process using one, two or three indicators.

In this article, we will present a mathematical model based on CC, which determines the stability of the technological process using the Kolmogorov and Smirnov criteria with one indicator. From the sample $X = (x_1, x_2, \dots, x_n)$ from X, we construct a variation series $X^* = (x^*{}_1, x^*{}_2, \dots, x^*{}_n)$. In this case, we come to test the following hypotheses at the $\propto$ value level:

$H_0: P(X < x) = F(x; \bar{X}_n; S_n^2)$;

$H_1: P(X < x) \neq F(x; \bar{X}_n; S_n^2)$.

here $Dn(\hat{\mu}_0, \hat{\sigma}_0^2) = \sqrt{n}\,\rho$, $\rho_t = \max_{1\le k\le n}\left|\Phi(Y_k^*) - \frac{2k-1}{2n}\right| + \frac{1}{2n}$, $Y_k^* = \frac{X_k^* - \bar{X}_n}{S_n}$

$\Phi(Y_k^*) \sim N(0,1)$ – standard normal distribution function.

Based on the above conclusion, H_0, is accepted in $Dn(\hat{\mu}_0, \hat{\sigma}_0^2) < k_{1-\propto}^c$ otherwise H_1.

In this case, the stability of TP depends on ρ, which represents the difference between the normal distribution and the empirical distribution, so we construct CC for this quantity.

Now, taking $g(X^*) = \rho$, we present the theorem for finding the upper control limit of ρ^c CC.

Theorem. $\propto = 0{,}01$ and $H_0: P(X < x) = F(x; \bar{X}_n; S_n^2)$ hypothesis is correct, then the controlled quantity for ρ^c-CC is ρ_t^c and $UCL_{\rho^c} = \frac{k_{0,99}^c}{\sqrt{n}}$ will be equal.

Explanation. If $X \sim N(\mu_0; \sigma_0^2)$, the upper control limit of ρ -NK is equal to:

$UCL = {k_{0,99}}/{\sqrt{n}}$. $k_{0,99}$ and $k_{0,99}^c$ are found in special tables [10, p. 347-348].

Result. If the inequality $\rho < {k_{0,99}^c}/{\sqrt{n}}$ or $\rho < {k_{0,99}}/{\sqrt{n}}$ is valid for samples at the specified unit times $t = 1,2,\dots,l$ then TP will be in a stable state.

Based on the result of the theorem, we present the ρ^c-CC model that determines the stability of TP.

ρ^c –CC mathematical model of TP stability.

If $X^* = (x^*{}_1, x^*{}_2, \dots, x^*{}_n)$ in unit $t = \overline{1;l}$ moments, the following relationship is valid: $\rho_t^c = \max\limits_{1\le k\le n}\left|\Phi(Y_{kt}^*) - \frac{2k-1}{2n}\right| + \frac{1}{2n} < \frac{k_{0,99}^c}{\sqrt{n}} = UCL_{\rho^c}$ then TP will be in a stable state.

$\rho^c - CC$ parameters of the mathematical model are found based on calculation methods. For example, the quantile of the Smirnov distribution $k_{0,99}^c$ is approximately found as the root of the equation, including $k_{0,99}^c$=1,035.

We will develop a numerical algorithm for generating this MM diagram and determining the stability of TP.

Determination of ρ^c CC threshold:

1) From the table of quantiles of the Smirnov distribution, we determine $k_{0,99}^c$and divide by $\sqrt{n}$ to determine UCL_{ρ^c} and draw the upper limit of CC in the Cartesian coordinate system;

Determining the values of ρ_t^c, which determine the stability of the technological process:

2). $t = \overline{1;l}$ $X = (x_1, x_2, \dots, x_n)$ taken from X at unit times $X^* = (x^*_1, x^*_2, \dots, x^*_n)$ we make variation series. We find the following middle value $\overline{X}_n = \frac{1}{n}\sum_{k=1}^{n} X^*$ and mean quadratic deviation

$$S_n = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(X_k - \bar{X}_n)^2}\,;$$

3) TP stability determinant $\rho_t^c = \max_{1\le k\le n}\left|\Phi(Y_{kt}^*) - \frac{2k-1}{2n}\right| + \frac{1}{2n}$ We calculate in unit times $t = \overline{1;l}$ and specify their values in the Cartesian coordinate system; Here $\Phi(Y_{kt}^*) = \frac{1}{2\pi}\int_0^{Y_{kt}^*} e^{-\frac{t^2}{2}}dt + \frac{1}{2}$.

Analysis of the stability of TP:

4). If the inequality $\rho_t^c < \frac{k_{0,99}^c}{\sqrt{n}}$ holds at unit times $= \overline{1;l}$, TJ is stable, otherwise it is unstable.

Below is a sample diagram of ρ^c- NK MM. In this case, the analysis of the stability of TP is presented in accordance with Figure 3.

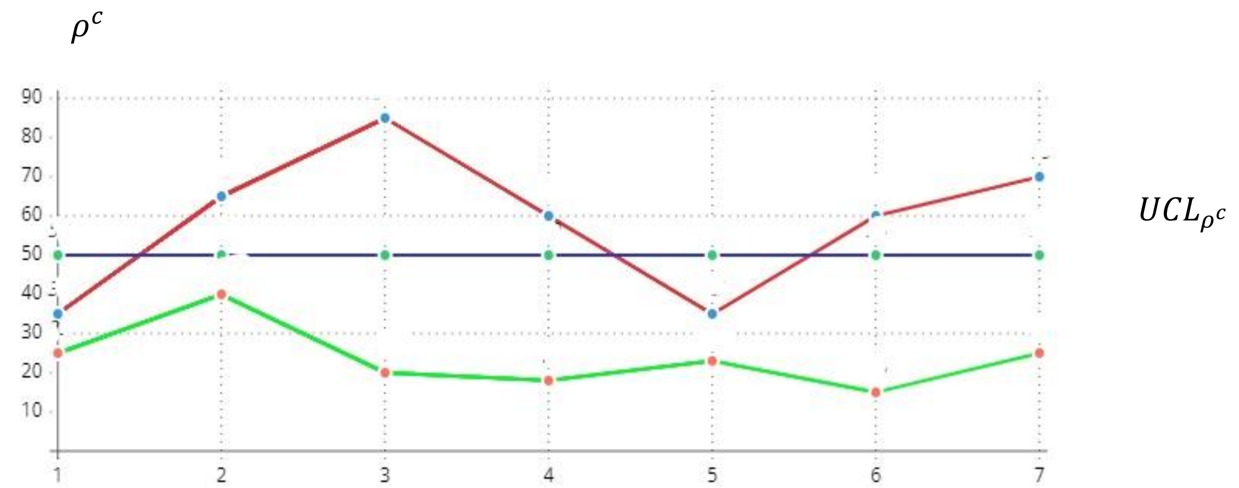


Figure 3. Mathematical model for determining the stability of TP.

a) green line - stable TP; b) red line - unstable TP.

Similarly, a mathematical model has been developed based on asymmetry and excess CCs, which determine the stability of the technological process with one indicator, and mathematical models based on CCs, which determine the stability of the technological process with two or three indicators using the Smirnov, Student, and Fisher criteria. Numerical algorithms have been developed and a software package has been created that simplifies the practical application of the developed mathematical models. Numerical algorithms and software packages were used to analyze some cases in TP. Based on the constructed mathematical models, cases were analyzed and a practical conclusion was drawn in the welding and painting workshops of the automobile manufacturing plant of JSC «UZ Auto Motors» for maintaining the filling between the machine panels and the thickness of the soil layer, as well as a case for assessing the current quality and quality control of the product output from the gilder machine.

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