CHEGARADA BUZILADIGAN 6-TARTIBLI DIFFERENSIAL TENGLAMALAR UCHUN GRIN FUNKSIYANI QURISH UCHUN MASALA.

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Masala. Quyidagi

$$L(y)=(x^{\alpha}\cdot y''')''',\ 0<\alpha\le 1 \qquad (1)$$

differensial operatorning

$$\begin{aligned} &y(0)=0 && y'''(1)=0\\ &y'(0)=0 && y^{(4)}(1)=0\\ &y''(0)=0 && y^{(5)}(1)=0 \end{aligned} \qquad (2)$$

chegaraviy shartlarni bajaradigan $y(x)$ ning Grin funksiyasi topilsin.

Yechish. Eng avval tenglamaning $L(y)=0$ umumiy yechimini topamiz va u quyidagi ko'rinishga ega:

$$y(x)=\frac{c_1x^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)}+\frac{c_2x^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)}+$$

$$\frac{c_3x^{3-\alpha}}{(1-\alpha)(2-\alpha)(3-\alpha)}+c_4\frac{x^2}{2}+c_5x+c_6 \qquad (3)$$

(3) dan foydalanib, umumiy yechimni chegaraviy shartlarga bo'ysundirsa,

$y(0)=0$ shartdan $c_6=0$,

$y'(0)=0$ shartdan $c_5=0$,

$y''(0)=0$ shartdan $c_4=0$,

$y'''(1)=0$ dan $\frac{1}{2}c_1+c_2+c_3=0$,

$y^{(4)}(1)=0$ dan $y^{(4)}(1)=\frac{1}{2}c_1(2-\alpha)+c_2(1-\alpha)-\alpha c_3=0,$

$y^{(5)}(1)=0$ dan $y^{(5)}(1)=\frac{1}{2}c_1(2-\alpha)(1-\alpha)+c_2(-\alpha)(1-\alpha)+\alpha(\alpha+1)c_3=0.$

Yuqoridagi 3-, 4-, 5- tenglamalarni tenglamalar sistemasi yordamida yechganimizda, $c_1=0,c_2=0,c_3=0$ lar ham kelib chiqadi. Natijada umumiy yechim trivial bo'lib qoladi, ya'ni $y(x)\equiv 0$ bo'ladi.

$\{(1),(2)\}$ masalaning yechimini qurish uchun oddiy Grin funksiyasi tuzamiz. $\{(1),(2)\}$ masalaning Grin funksiyasini quyidagi ko'rinishda izlaymiz:

$$G(x,s)=\begin{cases} A_1\dfrac{x^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)}+A_2\dfrac{x^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)}+ \\ +A_3\dfrac{x^{3-\alpha}}{(1-\alpha)(2-\alpha)(3-\alpha)}+A_4\dfrac{x^2}{2}+A_5x+A_6, & 0\le x\le s, \\ B_1\dfrac{x^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)}+B_2\dfrac{x^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)}+ \\ +B_3\dfrac{x^{3-\alpha}}{(1-\alpha)(2-\alpha)(3-\alpha)}+B_4\dfrac{x^2}{2}+B_5x+B_6, & s\le x\le 1, \end{cases} \quad (4)$$

Grin funksiyaning xossalariga ko‘ra quyidagilarni hosil qilamiz:

$G(0,s)=0,\ G_x'(0,s)=0,\ G_{xx}''(0,s)=0$ chegaraviy shartlardan $A_6=0$, $A_5=0$ va $A_4=0$ ekanligi kelib chiqadi.

$G_{xxx}'''(1,s)=0,\ G_{xxxx}^{(IV)}(1,s)=0,\ G_{xxxxx}^{(V)}(1,s)=0$ chegaraviy shartlardan esa $B_1=0$, $B_2=0$ va $B_3=0$ ekanligi kelib chiqadi.

Endi bu hadlarni qanday kelib chiqqanini ko‘rsak,

$$G(0,s)=\frac{A_1\cdot 0^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)}+\frac{A_2\cdot 0^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)}+$$

$$+\frac{A_3\cdot 0^{3-\alpha}}{(1-\alpha)(2-\alpha)(3-\alpha)}+A_4\frac{0^2}{2}+A_5 0+A_6=0$$

dan $A_6=0$ kelib chiqdi.

$$G_x'(0,s)=\frac{A_1\cdot 0^{4-\alpha}}{2(3-\alpha)(4-\alpha)}+\frac{A_2\cdot 0^{3-\alpha}}{(2-\alpha)(3-\alpha)}+\frac{A_3\cdot 0^{2-\alpha}}{(1-\alpha)(2-\alpha)}+A_4\cdot 0+A_5=0$$

yuqoridagi tenglamadan $A_5=0$ kelib chiqadi.

$$G_{xx}''(0,s)=\frac{A_1\cdot 0^{3-\alpha}}{2(3-\alpha)}+\frac{A_2\cdot 0^{2-\alpha}}{(2-\alpha)}+\frac{A_3\cdot 0^{1-\alpha}}{(1-\alpha)}+A_4=0$$

dan $A_4=0$ kelib chiqdi.

$$G_{xxx}'''(1,s)=\frac{B_1\cdot 1^{2-\alpha}}{2}+B_2\cdot 1^{1-\alpha}+B_3\cdot 1^{-\alpha}=0$$

$$G_{xxxx}^{(4)}(1,s)=\frac{B_1\cdot(2-\alpha)\cdot 1^{1-\alpha}}{2}+B_2\cdot(1-\alpha)-\alpha\cdot B_3=0$$

$$G_{xxxxx}^{(5)}(1,s)=\frac{1}{2}\cdot B_1\cdot(2-\alpha)\cdot(1-\alpha)+B_2\cdot(1-\alpha)\cdot(-\alpha)+B_3\alpha\cdot(1+\alpha)=0$$

tenglamalar sistemasini yechsak, $B_1=0$, $B_2=0$ va $B_3=0$ lar kelib chiqadi.

$$G(x,s)=\begin{cases}\dfrac{A_1x^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)}+\dfrac{A_2x^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)}+\\ +\dfrac{A_3x^{3-\alpha}}{(1-\alpha)(2-\alpha)(3-\alpha)}, & 0\le x\le s\\ B_4\dfrac{x^2}{2}+B_5x+B_6, & s\le x\le 1\end{cases}$$

yuqoridagi (4*) ko‘rinishga keladi. Endi noma’lum hadlarni topamiz. Buning uchun Grin funksiyaning xossalaridan, ya’ni shartlaridan foydalanamiz.

Uzluksizlik shartidan

$$\frac{A_1s^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)}+\frac{A_2s^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)}+\frac{A_3s^{3-\alpha}}{(1-\alpha)(2-\alpha)(3-\alpha)}=B_4\frac{s^2}{2}+B_5s+B_6,$$

(5)

1-tartibli hosila olib, tenglaymiz:

$$\frac{A_1s^{4-\alpha}}{2(3-\alpha)(4-\alpha)}+\frac{A_2s^{3-\alpha}}{(2-\alpha)(3-\alpha)}+\frac{A_3s^{2-\alpha}}{(1-\alpha)(2-\alpha)}=B_4s+B_5, \qquad (6)$$

2-tartibli hosila olib, tenglaymiz:

$$\frac{A_1s^{3-\alpha}}{2(3-\alpha)}+\frac{A_2s^{2-\alpha}}{(2-\alpha)}+\frac{A_3s^{1-\alpha}}{(1-\alpha)}=B_4, \qquad (7)$$

3-tartibli hosila olib tenglaymiz:

$$\frac{A_1s^{2-\alpha}}{2}+A_2s^{1-\alpha}+A_3s^{-\alpha}=0, \qquad (8)$$

4-tartibli hosilalar olib tenglaymiz:

$$\frac{A_1(2-\alpha)s^{1-\alpha}}{2}+A_2s^{-\alpha}(1-\alpha)+A_3s^{-\alpha-1}(-\alpha)=0, \qquad (9)$$

Endi 5-tartibli hosilalar olib $x=s$ uzilish ega ya’ni sakrashini tekshiramiz:

$$0-(\frac{1}{2}A_1(2-\alpha)(1-\alpha)s^{-\alpha}+A_2(1-\alpha)(-\alpha)s^{-\alpha-1}+A_3(1+\alpha)(-\alpha)s^{-\alpha-2})=s^{-\alpha}$$

(10)

yuqoridagi tenglikni $s^{-\alpha}$ ga bo‘lib, s ni darajadagi α larni yo‘qotib yuboramiz.

Xuddi shunday qilib, (8) va (9) tengliklarni ham $s^{-\alpha}$ ga bo‘lib yuboramiz.

Natijada,

$$\begin{cases}\dfrac{1}{2}A_1s^2+A_2s+A_3=0\\ \dfrac{1}{2}A_1(2-\alpha)s+A_2(1-\alpha)-A_3s^{-1}\alpha=0\\ \dfrac{1}{2}A_1(2-\alpha)(1-\alpha)+A_2(1-\alpha)\alpha s^{-1}+A_3(1+\alpha)\alpha s^{-2}=-1\end{cases}$$

Tenglamalar sistemasidan o‘rniga qo‘yish usuli yordamida A_1, A_2, A_3 hadlarni topib olamiz va quyidagi ko‘rinishga keladi:

$$A_1 = -\frac{1}{2\alpha^2 - 2\alpha + 1}$$

$$A_2 = \frac{s}{2\alpha^2 - 2\alpha + 1}$$

$$A_3 = -\frac{1}{2} \cdot \frac{s^2}{2\alpha^2 - 2\alpha + 1}$$

lar kelib chiqadi. Bu ma'lum hadlarni (5), (6) va (7) tengliklarga qo'yib noma'lum bo'lgan B_4, B_5 va B_6 larni ham topib olamiz:

$$B_4 = -\frac{1}{(1-\alpha)(2-\alpha)(3-\alpha)} \cdot \frac{s^{3-\alpha}}{2\alpha^2 - 2\alpha + 1},$$

$$B_5 = \frac{1}{(2-\alpha)(3-\alpha)(4-\alpha)} \cdot \frac{s^{4-\alpha}}{2\alpha^2 - 2\alpha + 1},$$

$$B_6 = \frac{-1}{2} \frac{1}{(3-\alpha)(4-\alpha)(5-\alpha)} \cdot \frac{s^{5-\alpha}}{2\alpha^2 - 2\alpha + 1}$$

Topilgan qiymatlarni (4*) ga qo'yib, Grin funksiyasini to'liq yozib olamiz:

$$G(x,s) = \begin{cases} \left(-\dfrac{x^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)} + \dfrac{s \cdot x^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)} - \dfrac{s^2 \cdot x^{3-\alpha}}{2(1-\alpha)(2-\alpha)(3-\alpha)} \right) \times \\ \times \dfrac{1}{2\alpha^2 - 2\alpha + 1}, & 0 \le x \le s, \\ \left(-\dfrac{x^2 \cdot s^{3-\alpha}}{2(1-\alpha)(2-\alpha)(3-\alpha)} + \dfrac{x \cdot s^{4-\alpha}}{(2-\alpha)(3-\alpha)(4-\alpha)} - \dfrac{s^{5-\alpha}}{2(3-\alpha)(4-\alpha)(5-\alpha)} \right) \times \\ \times \dfrac{1}{2\alpha^2 - 2\alpha + 1} & s \le x \le 1. \end{cases}$$

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