KASR TARTIBLI DIFFERENSIAL TENGLAMAGA QO'YILADIGAN BOSHLANG'ICH SHARTLI MASALA UCHUN EYLER METODI

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Annotatsiya: *Ushbu maqolada biz kasr tartibli differensial tenglamalar va ularning sistemasi uchun boshlang'ich shartli masalalarni yechishni Eyler hamda umulashgan Eyler usulidan foydalangan holda yoritishni hamda uning qiymatini hisoblash uchun Python dasturlash tilidagi algoritmdan foydalangan holda hisoblashni yoritib berganmiz. Yechim* α *ning turli qiymatlari uchun chizilgan.*

Kalit so'zlar: *Kaputo kasr tartibli hosila va integral, Eyler va umumlashgan Eyler metodi*

Hozirgi vaqtda kasr tartibli integral va hosilalar nazariyasi funksiyalar nazariyasining tarmoqlaridan biri bo'lib, muhim amaliy ahamiyatga ega bo'lmoqda [1-6]. Kasr tartibli hosila va integrallarga asoslangan matematik modellar matematik biologiyada [7], gidrogeologiyada issiqlik va massa almashinish jarayonlarini bir xil bo'lmagan muhitda modellashda [8-10], transonik oqimlarning elastoplastiklik muammolari, yarimo'tkazgichlar sohasidagi tadqiqotlar, epidemiologiyada, moliya va boshqa yo'nalishlarda e'tirof etiladi.

Kasr tartibli integrallar va hosilalar o'ziga xos xususiyatlarga ega va kasr integrallari va hosilalari bilan tenglamalarni echishning analitik usullari har doim ham samarali emas. So'nggi yillarda sonli usullar keng rivojlandi. Biroq, ushbu yo'nalishda erishilgan muvaffaqiyatlarga qaramay, shunga o'xshash muammolarning umumiy sinfi uchun taqribiy usullardan foydalanishni nazariy asoslash masalasi ochiqligicha qolmoqda.

Aytaylik $\alpha>0$ va $n=\alpha$ bo'lsin. Kaputo α kasr tartibli hosila quyidagicha ta'riflanadi:

$$ {}_a^C D_t^\alpha f(t)=\frac{1}{\Gamma(n-\alpha)}\int_a^t (t-u)^{n-\alpha-1}\left(\frac{d}{du}\right)^n f(u)\,du $$

$a=0$ uchun, quyidagicha belgilash kiritamiz:

$$ {}^C D_t^\alpha f(t)=D^\alpha f(t). $$

Endi kasr tartibli differensial tenglamalar uchun umumlashga Eyler metodi bilan tanishamiz:

$D^{\alpha} y(t) = f(t, y(t))$, $y(t_0) = y_0$, bu yerda $0 < \alpha \le 1$.

$[t_0, a]$ kesmada $y(t)$, $D^{\alpha} y(t)$, $D^{2\alpha} y(t)$ uzluksiz deb qaraymiz va umumlashgan Teylor formulasini qo'llaymiz:

$$y(t) = y(t_0) + D^{\alpha} y(t_0) \frac{t^{\alpha}}{\Gamma(\alpha+1)} + D^{2\alpha}(\eta) \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}, \ t_0 < \eta \le t.$$

Iterativ Eyler formulasini hosil qilamiz:

$$y(t) \approx y(t_0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_0, y(t_0)),$$

yoki rekurrent ko'rinishda ifodalasak:

$$y_{n+1} \approx y_n + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_n, y_n).$$

bu yerda h t vaqt o'qi bo'yicha qadamni bildiradi.

1-misol. Quyidagi kasr tartibli differensial tenglamani yeching:

$$D^{\frac{1}{2}} y(t) = y(t) - \frac{2t}{y(t)},$$

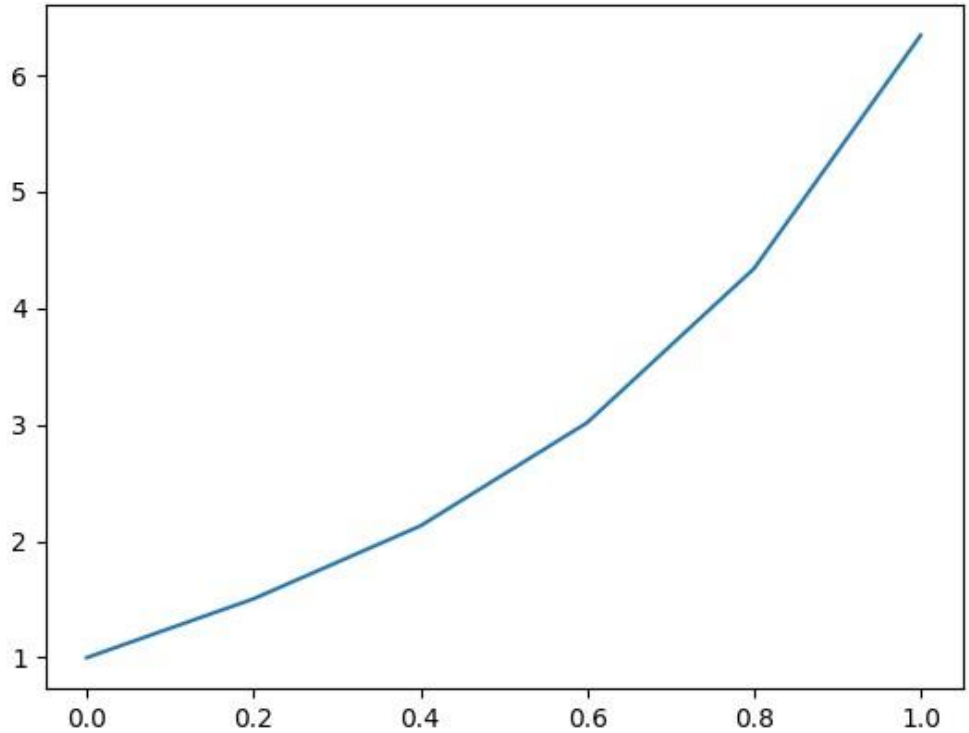
$y(0) = 1$ boshlang'ich shart,

bu yerda $t \in [0,1]$ va $h = 0.2$ qadam.

Yechish. Eyler metodidan foydalanib yuqoridagi misolni taqribiy qiymatlarini topuvchi python dasturlash tilidagi algoritmni quramiz:

```
import numpy as np
import math
import matplotlib.pyplot as plt
from scipy.special import gamma, factorial
h=0.2
a=0.5
t=np.arange(0,1.1,0.2)
y=[1]*6
def f(t, y):
return y-2*t/y
for i in range (5):
y[i+1]=y[i]+math.pow(h,a)/gamma(a+1)*f(t[i],y[i])
plt.plot(t,y)
```

plt.show
y
0 1,
0.2 1.504626504404032,
0.4 2.1297476237842687,
0.6 3.014921181437984,
0.8 4.33547869767681,
1 6.3370447256253986



1-rasm. 1-misoldagi $y(t)$ yechim grafigi

2-misol. Eyler metodi yordamida quyidagi kasr tartibli differensial tenglamani tyaqirbiy yeching:

$$D^{\alpha} y(t) = \frac{y(t)-t}{y(t)+t},$$

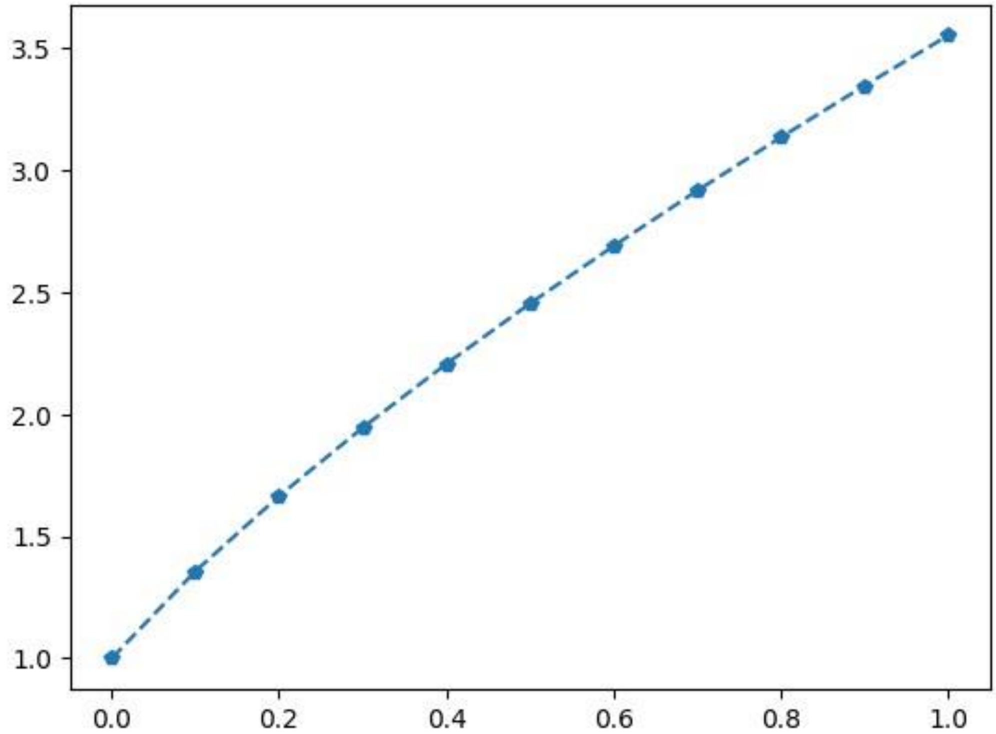
$y(0)=1$ boshlang'ich shart, $h=0.1$ qadam va $t\in[0,1]$.

Yechish. Eyler metodidan foydalanib yuqoridagi misolni taqribiy qiymatlarini topuvchi python dasturlash tilidagi algoritmni quramiz:

```
import numpy as np
import math
import matplotlib.pyplot as plt
```

```
from scipy.special import gamma, factorial
h=0.1
a=0.5
t=np.arange(0,1.1,h)
y=[1]*len(t)
def f(t, y):
return (y-t)/(y+t)
for i in range (len(t)-1):
y[i+1]=y[i]+math.pow(h,a)/gamma(a+1)*f(t[i],y[i])
plt.plot(t,y,'p--')
plt.show
y
```

1	1,
0.1	1.3568248232305542,
0.2	1.6646629985758419,
0.3	1.9449431971738307,
0.4	2.206400398538559,
0.5	2.4537025794519525,
0.6	2.6897214604346718,
0.7	2.91638639231075,
0.8	3.1350747726034194,
0.9	3.3468147498880425,
1	3.5524004163289273



2-rasm. 2-misoldagi $y(t)$ yechim grafigi

3-misol. Kasr tartibli differensial tenglamalar sistemasini taqribiy yeching:

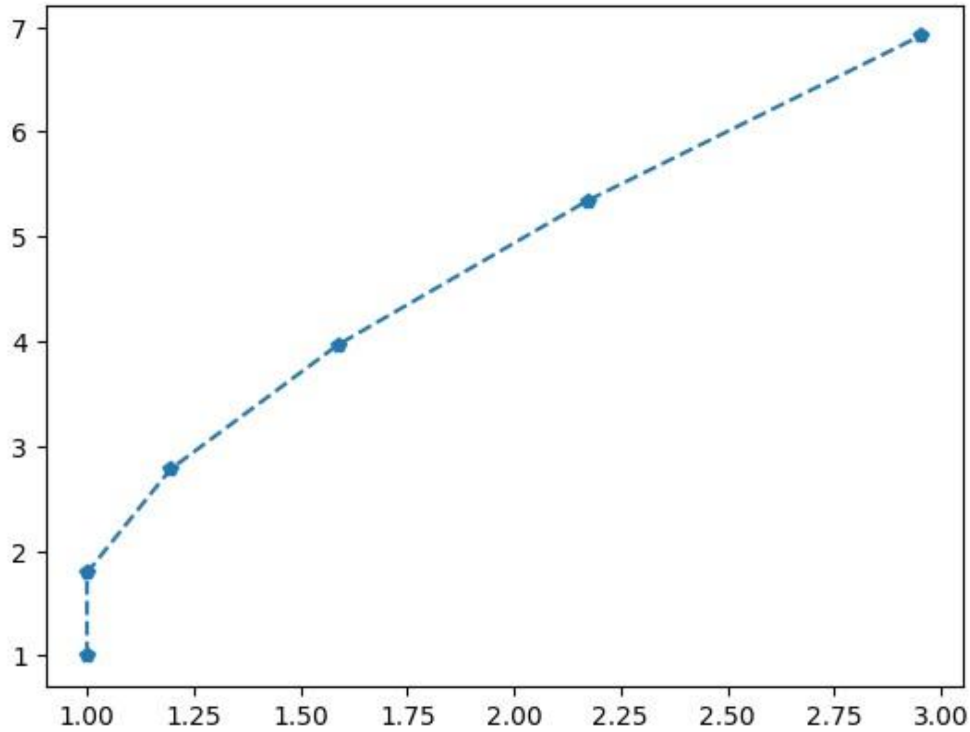
$$\begin{cases} D^{\frac{1}{2}}x(t)=\dfrac{y(t)-t}{t} \\ D^{\frac{1}{2}}y(t)=\dfrac{x(t)+t}{t} \end{cases},$$

$x(1)=1$ va $y(1)=1$ boshlang'ich shartlar, $h=0.2$ vaqt qadami va $t\in[1,2]$.

Yechish. Eyler metodidan foydalanib yuqoridagi misolni taqribiy qiymatlarini topuvchi python dasturlash tilidagi algoritmni quramiz:

```
import numpy as np
import math
import matplotlib.pyplot as plt
from scipy.special import gamma, factorial
def f(t,x,y):
return (y-t)/t
def g(t,x,y):
return (y+t)/t
d=0.2
a=0.5
```

```
h=math.pow(d,a)*gamma(a+1)
t=np.arange(1,2.1,d)
x=[1]*len(t)
y=[1]*len(t)
for i in range (len(t)-1):
x[i+1]=x[i]+h*f(t[i],x[i],y[i])
y[i+1]=y[i]+h*g(t[i],x[i],y[i])
plt.plot(x,y,'p--')
plt.show
```



Eslatma: Belgilashdan foydalanib

$k_1 = f\left(t_n, y_n\right),$

$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right),$

umumiyroq Eyler iterativ formulasini quyidagi ko'rinishda yozishimiz mumkin:

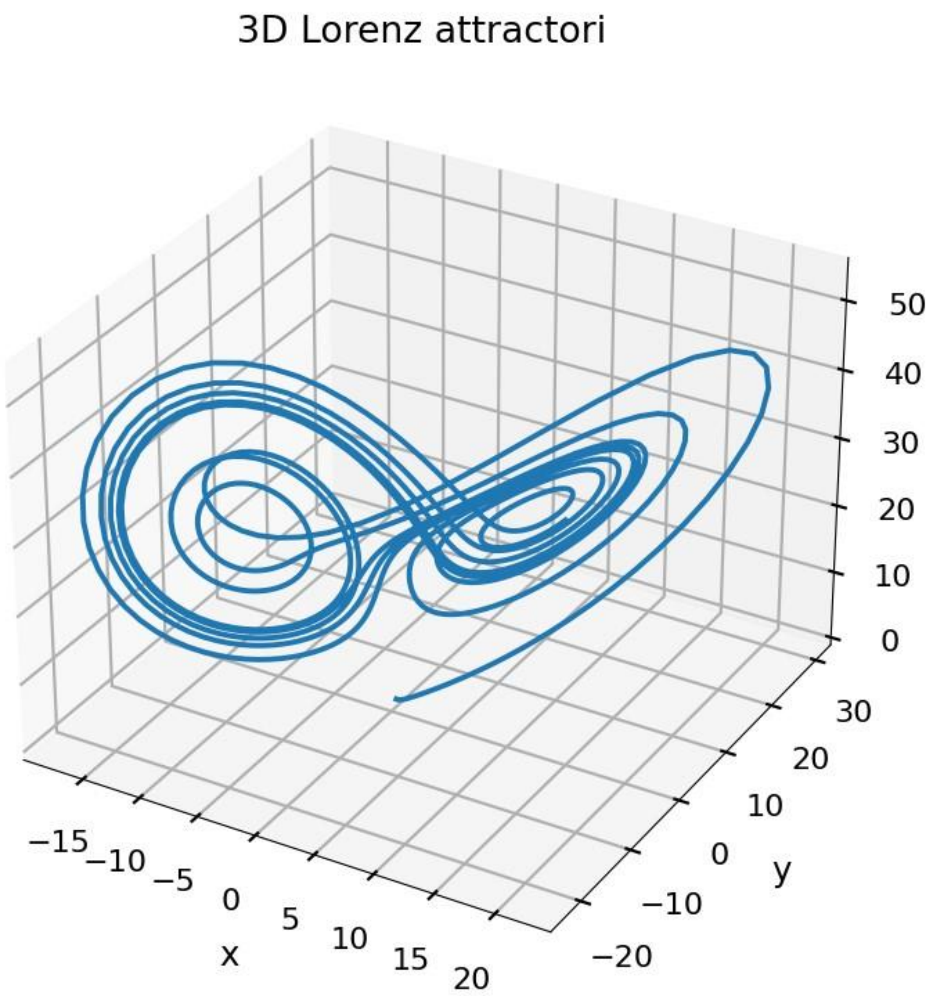
$y_{n+1} = y_n + hk_2.$

4-misol. (Lorenz Attractor) Lorenz Attractor sistemasini yeching:

$$\begin{cases} D^{0.98}x(t) = -10(x-y),\ x(0)=0 \\ D^{0.98}y(t) = 28x - y - xz,\ \ y(0)=1 \\ D^{0.98}z(t) = xy - \dfrac{8}{3}z,\ \ z(0)=0 \end{cases}$$

Yechish. Umumiyroq Eyler iterativ formulasi foydalanib yuqoridagi misolni taqribiy qiymatlarini topuvchi python dasturlash tilidagi algoritmni quramiz:

```
import numpy as np
from mpl_toolkits import mplot3d
import math
import matplotlib.pyplot as plt
from scipy.special import gamma, factorial
plt.style.use('seaborn-poster')
ax = plt.axes(projection='3d')
def f(t,x,y,z):
return -10*(x-y)
def g(t,x,y,z):
return 28*x-y-x*z
def k(t,x,y,z):
return x*y-8/3*z
d=0.01
a=0.98
h=math.pow(d,a)*gamma(a+1)
t=np.arange(0,10.1,d)
x=[0]*len(t)
y=[1]*len(t)
z=[0]*len(t)
for i in range (len(t)-1):
x[i+1]=x[i]+h*f(t[i],x[i],y[i],z[i])
y[i+1]=y[i]+h*g(t[i],x[i],y[i],z[i])
z[i+1]=z[i]+h*k(t[i],x[i],y[i],z[i])
ax.plot3D(x, y, z)
ax.set_title('3D Parametric Plot')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
plt.show
```



4-rasm. 3D Lorenz attractori yechimi $\alpha = 0.98$ uchun

Yuqoridagi kabi masalalar [6-10] ishlarda ham ko‘rilgan.

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