THEOREM ABOUT THE SUM OF THE GALTON-WATSON PROCESS IN ALL PERIODS

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Abstract: *Sum of the total number of particles up to time n for branching Galton-Watson processes. This article considers limit theorems for sums of the total number of particles for branching processes.*

Key words: *The sum of Galton-Watson process, Markov branching chain, The generating function, The random variable, The distribution function.*

Let's suppose that the random variable μ_{n+1} Consists of the sum of μ_1 $\mu_{1,n+1}^{(j)}$, $j=\overline{1,\mu_1}$ Random variables $\mu_{1,n+1}^{(j)}$ Is independent of j and identically distributed with μ_{n+1} , so

$$\mu_{n+1}=\begin{cases}\mu_{1,n+1}^{(1)}+\mu_{1,n+1}^{(2)}+\cdots+\mu_{1,n+1}^{(\mu_1)}, & \mu_1>0\\ 0, & \mu_1=0\end{cases}$$

Where $\mu_n^{(j)}$ Is independent and identically distributed with μ_n [5].

If $f_1(s)$ is defined as the generating function of $MS^{\mu_1},(S\in[0,1])$ μ_1 Then , taking into account above , the generating function of μ_n Is equal to

$$f_n(s)=f_1\underbrace{f(f\ldots f_n(s))\ldots)}_{n-1}=f_k(f_{n-k}(s))=f_{n-k}(f_k(s)),$$

$$f_0(s)=s, f_n(s)=MS^{\mu_n},$$

μ_n n- the number of particles.

If we donate by

$$\mu_0+\mu_1+\cdots+\mu_n+\cdots=\mu \qquad (1)$$

The number of particles in all periods , then the generating function of (1) is equal to $MS^{\mu}=F(S), F(1)=1$ or $F(1)<1$, that is , μ can be finite or infinite. In particular, if we donate by $F_n(S)$ The generating function of $\mu_0+\mu_1+\cdots+\mu_n$, then the relation

$$F_{n+1}(S)=Sf(F_n(S)) \qquad (2)$$

Is indicated, where $f(s)=f_1(s)$, as a result, the generating function of (1) Is equal to $F(S)=Sf(F(S))$ In $n\to+\infty$ (Hawkins, Ulam, Good, Otter,Harris [2]).

μ can be learned by using (1) and (2) .

Works that were considered important results for (1) by Duoss, O,V,Viskov, Boyd [3] can be cited [2]:

$P\{ \mu_0 + \mu_1 + \cdots + \mu_n + \cdots = J/\mu_0 = i\} = ij^{-1}p(\xi_1 + \xi_2 + \cdots + \xi_j = j - i)$ (3)

Where, ξ_i Is independent and identically distributed with μ_1 . This theorem is very important for the critical and subcritical cases , and for the supercritical case j goes to the infinity.

It is known that studying μ in (3) leads to studying $\mu_1^{(1)} + \mu_1^{(2)} + \cdots + \mu_1^{(j)}$, where , $\mu_1^{(k)}, k = \overline{\overline{i,j}}$ Is independent and identically distributed with μ_1. This in turn leads to the study of a sums of independent and lattice identically distributed random variables of $x_1, x_2, \ldots, x_n$.

It is known that $S_n = x_1 + x_2 + \cdots + x_n$ Is also lattice distributed and the $P_n(k) = P(S_n = na + kh)$ probability is studied, where h is a maximum step, a is a real number.

If we enter symbols $Z_{n,k} = \frac{an+kh-A_n}{B_n}$, $A_n = MS_n, B_n^2 = DS_n$

Then in $0 < Dx_I < +\infty$ and $n \to \infty$

$$\frac{B_n}{h} P_n(k) - \frac{1}{\sqrt{2\pi}} e^{-\frac{Z_{n,k}^2}{2}} \to 0 \qquad (4)$$

Relation is evenly performed with respect to k [4]

Theorem 1. If μ_1 Is a Poisson distributed branching process with maximum step h , then under the condition $M\mu_1 \leq 1, D\mu_1 < +\infty$,

An equivalent theorem to (4) can be given for

$P(\mu_1^{(1)} + \mu_1^{(2)} + \cdots + \mu_1^{(j)} = aj + kh)$, and the proof of this theorem is done as (4).

If μ_1 Branching process has a distribution function and

In relation $M\mu_1 \leq 1, D\mu_1 < +\infty$, if the relation

$$\frac{\mu_1^{(1)} + \mu_1^{(2)} + \cdots + \mu_1^{(j)} - JM\mu_1}{\sqrt{jD\mu_1}}$$

Has a density function $P_J(x)$ Then in order for $j \to \infty$ da

$$P_J(x) - \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \to 0 \qquad (5)$$

To be valid in $j \to \infty$, it is necessary to have a n_0 Satisfying $P_{n_0}(x) < \infty$,

Where $\mu_1^{(k)}, k = \overline{\overline{i,j}}$ Are independent and identically distributed ,having the distribution as μ_1 .

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