MURAKKAB ARALASH TURDAGI UCHINCHI TARTIBLI TENGLAMA UCHUN BIR CHEGARAVIY MASALANING YAGONALIGI .

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Annotatsiya. Mazkur *maqolada chegaralanmagan sohada uchinchi tartibli aralash tipdagi tenglama uchun chegaraviy masala qaralgan bo'lib,masalaning yechimining yagonaligi integral energiya usulida isbotlangan.*

Kalit so`zlar:*Yuqori tartibli tenglama, Gauss-Ostrogradskiy formulasi,yechimning yagonaligi.*

Rivojlanayotgan hamda izlanishlarga boy matematika olamida ikkinchi,uchinchi va yuqori tartibli tenglamalar uchun ko'plab olimlar tomonidan chegaraviy masalalar tahlil etilgan. Murakkab va murakkab-aralash tipdagi tenglamalar uchun ustozlarimiz M.S.Salohiddinov [2], T.D.Jo'rayev [1] va ularning shogirdlari tomonidan chegaraviy masalalar qo'yilib ularni o'rganish nazariyalari yaratilgan.Bu maqolada uchinchi tartibli tenglama uchun chegaralanmagan sohada chegaraviy masala o'rganilgan.

Masalaning qo'yilishi:

Ushbu

$$\begin{cases} \dfrac{\partial}{\partial x}(U_{xx}+U_{yy})+C(x,y)U(x,y)=0, y>0 \\ \dfrac{\partial}{\partial x}(U_{xx}-U_{yy})=0, y<0 \end{cases} \quad (1)$$

tenglamani $D=\{D_1 \cup D_2 \cup y=0\}$ sohada qaraymiz.

Bunda $D_1=\{(x,y): x>0, y>0\}$ $D_2=\{(x,y): x>0, y>-x\}$, $C(x,y)$ berilgan funksiya.

Masala. D sohada (1) tenglamaning quyidagi shartlarni qanoatlantiruvchi $U(x,y)$ yechimi topilsin:

1) $U(x,y)$ funksiya $\bar{D}$ da uzluksiz;

2) $U(x,y)$ D sohada (1) tenglamaning regulyar yechimi, $y \neq 0$;

3) $U(x,y)$ quyidagi chegaraviy shartlarni qanoatlantirsin:

$$U(0,y)=\varphi_1(y), \quad 0 \le y < \infty \quad (2)$$

$$U_x(0,y)=\varphi_2(y), \quad 0 \le y < \infty \quad (3)$$

$$U(x,-x)=\psi_1(x), \quad 0\le x<\infty \tag{4}$$

$$\frac{\partial U(x,-x)}{\partial n}=\psi_2(x), \quad 0<x<\infty \tag{5}$$

$$\lim_{R\to\infty} U_x=0, \quad R^2=x^2+y^2, \quad x>0, \quad y>0 \tag{6}$$

bu yerda $\varphi_i(y),\psi_i(x),\ (i=1,2)$ berilgan funksiyalar bo‘lib, $\varphi_1(0)=\psi_1(0)$ kelishuv shartini qanoatlantiradi.

$$\frac{\partial}{\partial x}U=V \tag{7}$$

deb belgilash kiritsa, u holda (1) tenglama

$$\begin{cases}\Delta V+CU=0\\ \square V=0\end{cases} \tag{8}$$

ko‘rinishni oladi, bunda

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-elliptik\ \ operator,$$

$$\square=\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-giperbolik\ \ operator.$$

(2)-(6) chegaraviy shartlardan foydalanib noma’lum $V(x,y)$ funksiya uchun quyidagi chegaraviy shartlarni olamiz;

$$V(x,y)|_{y=-x}=\frac{1}{2}\left(\psi_1'(x)+\sqrt{2}\psi_2(x)\right)=\psi(x)\,, \tag{9}$$

$$V(x,y)|_{x=0}=\varphi_2(y)\,, \tag{10}$$

$$\lim_{R\to\infty}V(x,y)=0\,. \tag{11}$$

Masala yechimining yagonaligi

Teorema. Agar (1) – (6) masala yechimga ega bo‘lsa va ushbu

a) $|C(x,y)|\le\frac{N_1}{R}\,,R\to\infty\,,N_1=const$

b) $C_x(x,y)\ge 0$

shartlar bajarilsa, u holda masala yechimi yagona bo‘ladi.

Isbot: Qo‘yilgan masala yechimining yagonaligini ko‘rsatish uchun bir jinsli masalani trivial yechimga ega ekanligini ko‘rsatish yetarli. Shuning uchun

$$\varphi_i(x)=\varphi_i(y)=0\ (i=1.2) \tag{12}$$

bo’lsin. U holda (7) belgilashga ko‘ra masala quyidagi ko‘rinishga keladi

$$\begin{cases} \Delta V + CU = 0 \\ \square V = 0 \end{cases} \qquad (8)$$

$$V(x,y)|_{y=-x} = 0, \qquad (13)$$

$$V(x,y)|_{x=0} = 0. \qquad (14)$$

Masala qaralayotgan D_1 sohani R radiusli aylana bilan, D_2 ni esa $x-y=R$ radiusli chiziq bilan chegaralaymiz, ya'ni $D_{1R}=\{(x,y): x^2+y^2<R^2,\ \ x>0,\ \ y>0\}$, D_{2R} esa $AC: x+y=0$ va $BC: x-y=R$ chiziqlar bilan chegaralangan soha.

D_{1R} sohaning chegarasini $\sigma_R=\{(x,y): x^2+y^2=R^2, x>0, y>0\}$, $A(0,0)B(R,0)$ va $D(0,R)$ $A(0,0)$ chiziqlar bilan belgilaymiz.

(8) ning birinchi tenglamasini $V(x,y)$ funksiyaga koʻpaytirib, D_{1R} soha boʻyicha integral olamiz

$$\iint_{D_{1R}} V\left[(V_{xx}+V_{yy})+CU\right]dxdy=0. \qquad (15)$$

Bu tenglikni quyidagi koʻrinishda yozib olamiz

$$\iint_{D_{1R}}\left[(VV_x)_x+(VV_y)_y-(V_x)^2-(V_y)^2+\frac{1}{2}(CU^2)_x-\frac{1}{2}C_xU^2\right]dxdy=0. \qquad (16)$$

Oxirgi (16) tenglikka Gauss-Ostrogradskiy formulasini qoʻllab [3],

$$\int_{\partial D_{1R}} V\left[V_x\cos(n,x)+V_y\cos(n,y)\right]ds+\frac{1}{2}\int_{\partial D_{1R}} CU^2dy=\iint_{D_{1R}}\left[V_x^2+V_y^2+C_xU^2\right]dxdy$$

yoki

$$\int_{AB} V\frac{dV}{dn}ds+\int_{G_R} V\frac{dV}{dn}ds+\int_{DA} V\frac{dV}{dn}ds+\frac{1}{2}\int_{AB} CU^2dy+$$

$$+\frac{1}{2}\int_{G_R} CU^2dy+\frac{1}{2}\int_{DA} CU^2dy=\iint_{D_{1R}}\left[V_x^2+V_y^2+C_xU^2\right]dxdy$$

koʻrinishidagi ifodaga kelamiz. Bir jinsli (13) va (14) shartlardan hamda AB chiziq ustiga $dy=0$ ekanligidan foydalansak quyidagiga ega boʻlamiz

$$\int_{G_R} V\frac{\partial V}{\partial n}ds-\int_{AB} V(x,0)V_y(x,0)dx+\frac{1}{2}\int_{G_R} CU^2dy=\iint_{D_{1R}}\left[V_x^2+V_y^2+C_xU^2\right]dxdy \qquad (17)$$

Endi (8) ning ikkinchi tenglamasini $V(x,y)$ ga koʻpaytirib, D_{2R} soha boʻyicha integrallaymiz

$$\iint_{D_{2R}} V(V_{xx}-V_{yy})dxdy=0. \qquad (18)$$

Bu tenglikni quyidagi koʻrinishda yozib olamiz:

$$\iint_{D_{2R}}\left[(VV_x)_x-(VV_y)_y-(V_x)^2+(V_y)^2\right]dxdy=0. \qquad (19)$$

(19) tenglikka Gauss-Ostrogradskiy formulasini qoʻllasak ushbu ifodani olamiz

$$\int_{\partial D_{2R}} V\left[V_x\cos(n,x)+V_y\cos(n,y)\right]ds=\iint_{D_{2R}}\left[V_x^2-V_y^2\right]dxdy$$

yoki

$$\int_{AC} VV_xdy+VV_ydx+\int_{CB} VV_xdy+VV_ydx+\int_{BA} VV_xdy+VV_ydx=\iint_{D_{2R}}\left[V_x^2-V_y^2\right]dxdy. \qquad (20)$$

Bir jinsli (13) va (14) shartlardan hamda AC chiziq ustida $dx=-dy$, CB chiziq ustida $dx=dy$ ekanligidan foydalanib (20) tenglikning chap tomonini hisoblaymiz

$$\int_{CB} VV_xdy+VV_ydx=\int_{CB} V\left(V_xdy+V_ydx\right)=\int_{CB} V\left(V_xdx+V_ydy\right)=\frac{1}{2}V^2(B).$$

Buni (20) ga qo‘ysak

$$\int_{BA} VV_ydx+\frac{1}{2}V^2(B)=\iint_{D_{2R}}\left[V_x^2-V_y^2\right]dxdy \qquad (21)$$

tenglik hosil bo‘ladi.

Endi (21) tenglikni o‘ng tomonini hisoblaymiz. Bu ikki karrali integralni hisoblash uchun yangi harakteristik koordinatalar sistemasiga o‘tamiz. U holda D_{2R} soha A_1C_1, C_1B_1, B_1A_1 tomonli yangi sohaga o‘tadi. Bular mos holda $\eta=0,\ \xi=1$ va $\eta=\xi$ chiziqlarda yotuvchi kesmalardir. $\xi=x-y,\ \eta=x+y$ belgilashlarga ko‘ra $x=\frac{\xi+\eta}{2},\ y=\frac{\eta-\xi}{2},\ V_x=V_\xi+V_\eta,\ V_y=-V_\xi+V_\eta$ tengliklar o‘rinli bo‘ladi.Bunga asosan (21) dan

$$\iint_{D_{2R}}\left[V_x^2-V_y^2\right]dxdy=\frac{1}{4}\iint_{D'_{2R}}\left[(V_\xi+V_\eta)^2-(-V_\xi+V_\eta)^2\right]d\xi d\eta=\frac{1}{4}\iint_{D'_{2R}}\left[V_\xi^2+V_\eta^2-V_\xi^2-V_\eta^2+4V_\xi V_\eta\right]d\xi d\eta=$$

$$=\frac{1}{4}\iint_{D'_{2R}}4V_\xi V_\eta d\xi d\eta=\iint_{D'_{2R}}\frac{\partial}{\partial\eta}(VV_\xi)d\xi d\eta-\iint_{D'_{2R}}VV_{\xi\eta}d\xi d\eta=\iint_{D'_{2R}}\frac{\partial}{\partial\eta}(VV_\xi)d\xi d\eta=-\int_{A_1C_1+C_1B_1+B_1A_1}VV_\xi d\xi$$

ifodaga ega bo‘lamiz. Bu yerda C_1B_1 chiziqda $d\xi=0$ ekanligini e’tiborga olsak

$$\iint_{D_{2R}}\left[V_x^2-V_y^2\right]dxdy=-\int_{A_1C_1+B_1A_1}VV_\xi d\xi$$

tenglikka ega bo‘lamiz. (13) va (14) shartlardan foydalanamiz.

$$\int_{A_1C_1}VV_\xi d\xi=\int_{A_1C_1}\left(\frac{1}{2}V^2\right)_\xi=\frac{1}{2}V^2\Big|_{A_1}^{C_1}=\frac{1}{2}V^2(C_1)=0,$$

$$\int_{B_1A_1}VV_\xi d\xi=\int_{B_1A_1}\left(\frac{1}{2}V^2\right)_\xi d\xi=\frac{1}{2}V^2\Big|_{B_1}^{A_1}=-\frac{1}{2}V^2(B_1).$$

Bulardan esa

$$\iint_{D_{2R}}\left[V_x^2-V_y^2\right]dxdy=-\frac{1}{2}V^2(B) \qquad (22)$$

hosil bo‘ladi. (22) ni (21) ga olib borib qo‘ysak va

$V(x,0)=\tau_1(x)$

$V_y(x,0)=\nu_1(x)$

belgilashni kiritsak, quyidagiga ega bo'lamiz

$$\int_{BA} VV_y dx=\int_B^A \tau_1(x)\nu_1(x)dx=\frac{1}{2}V^2(B)-\frac{1}{2}V^2(B)=0.$$

Bizga

$$\int_0^B \tau_1(x)\nu_1(x)dx=0$$

ekanligi ma'lum. (11) ga asosan $R\to\infty$ da limitga o'tamiz. Bundan esa

$$\int_0^\infty \tau_1(x)\nu_1(x)dx=0 \qquad (23)$$

ekanligi kelib chiqadi.(23) tenglikni (17)ga qo'yamiz va $R\to\infty$ da limitga o'tamiz. U holda teorema shartiga va (14) ga ko'ra

$$\iint_{D_1}\left[V_x^2+V_y^2\right]dxdy+\iint_{D_1} C_x U^2 dxdy=0$$

tenglik o'rinli bo'ladi.

Agar $C_x=0$ bo'lsa, so'nggi tenglikdan $V_x=0,\ \ V_y=0$ ekanligi kelib chiqadi. Bundan esa $V=const.$ $V(0,y)=0$ shartga ko'ra $V=U_x=0$ ga ega bo'lamiz. Bu tenglamani $U(0,y)=0$ shart ostida yechsak, $\bar{D}_1$ sohada $U\equiv 0$ ekanligi kelib chiqadi.

Agar $C_x\neq 0$ bo'lsa, yuqoridagi tenglikdan D_1 sohada $U\equiv 0$ bo'ladi. Funksiyaning yopiq sohada uzluksizligidan $\bar{D}_1$ sohada $U\equiv 0$ ayniyatni olamiz. $\bar{D}_1$ sohada $V\equiv 0$ ekanligidan $\tau_1\equiv 0$ tenglikka ega bo'lamiz.

D_2 sohada (8) ning ikkinchi tenglamasi uchun Koshi masalasining yechimi yagonaligidan $V\equiv 0$ ekanligi kelib chiqadi

$V=U_x=0$.

Tenglamani $U(x,-x)=0$ shart bilan yechsak, $\bar{D}_2$ da $U\equiv 0$ trivial yechimga ega ekanligini ko'rsatish mumkin. Yechimning yagonaligi isbotlandi.

FOYDALANILGAN ADABIYOTLAR:

1. **Джураев Т.Д.** Краевые задачи для уравнения смешанного и смешанно-составного типов. Ташкент. Фан. 1979.-240 с.

2. **Салахитдинов М.С.** Уравнения смешанно-составного типа. Ташкент. Фан.1974.-156с.

3. **Тихонов А.Н. , Самарский А.А.** Уравнения математической физики. М. Наука 1977.-736с.