"FUNDAMENTALS OF GEOMETRY" IN HIGHER EDUCATION

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Methods and tools for teaching mathematics with other topics at higher education institutions have advanced in modern times. "Foundations of Geometry" is a subject taught in mathematics education departments of higher education institutions. This topic has evolved as the scientific foundation for non-Euclidean geometry. Furthermore, geometry is an ancient topic and is regarded as the first scientific subject taught to school children in its entirety. This is because, in geometry studies, pupils encounter scientific ideas such as "axioms," "theorems," and "proofs" for the first time. This geometry-specific condition is also pertinent to the subject of "Foundations of Geometry" in higher education institutions.

Understanding the subject of "Foundations of Geometry" is considered vital information for people who have earned skills in any branch of mathematics in the twenty-first century. Learning how Euclidean geometry was built in Euclid's book "Elements" was a huge feat for any aficionado, and the subject of "Foundations of Geometry" is currently regarded as a significant field for mathematicians. In the nineteenth and twentieth centuries, revolutionary changes occurred in every field. These transformations also caused changes in geometry. In addition to different non-Euclidean geometries, topology, theory of complexity, integral geometry, geometric calibration, and even discrete mathematics, often known as discrete geometry, have emerged in the modern era.

The projective metric geometry, defined by Keli and Kleyn in the projective metrical phases known as the "Erlangen program," is thought to be a scientific path influenced by V.I. Lobachevsky's idea [1]. The number of n-dimensional projective metric geometries, according to the Keli-Kleyn theory, is 3^n [2]. So, in the plane, there are nine various geometries, one of which is Lobachevsky's geometry, which created alterations in the perspective of geometry and differs from Euclidean geometry, as well as seven different non-Euclidean geometries . However, when the dimension of the phase space is greater than two, it is not difficult to imagine that the number of projective metrics in phase geometry becomes numerous.

The emergence of the concept of "manifolds" in the field, as well as the development of the topology section, which studies the characteristics of geometric shapes in continuous deformation, contributed to the creation of "manifolds" in various fields, not only in the field of mathematical sciences, but also in other specific sciences, and are widely used. The development of manifold theory and its application in numerous sectors has resulted in many results relating to this subject. However, according to V. Nesh's theory, any simple manifold of a large size can be created in the Euclidean phase. This is because constructing original manifolds is a scientific direction coming from the generalization of boundary theory in geometry. The theory of semi-Euclidean phases originated near the beginning of the twenty-first century, leading to the development of elliptic and hyperbolic phases and related geometries [3].

Recent developments in discrete mathematics, known as discrete geometry with graph theory, have given rise to a new path in geometry that is frequently used in current technology. These orientations, however, do not reflect the modern developments occurring in geometry as a whole, both in terms of the topic itself and its programs. Unfortunately, the subject "Fundamentals of Geometry," which was developed in the early twentieth century, has been retained without changing its essence up to the present day.

The paper by J. Buda, titled "Integrating Non-Euclidean Geometry into High School" [4], explores the incorporation of non-Euclidean geometry into high school curricula. Furthermore, Judith N. Cederberg's publication, "A Course in Modern Geometries" [5], in conjunction with Francesco C., Boccaletti D., Roberto C's "The Mathematics of Minkowski Space-Time: With an Introduction to Commutative Hypercomplex Numbers" [6], Wylie C. R's "Foundations of Geometry" [7], Audun H's "Geometry: Our Cultural Heritage" [8], and Vincent A., Athanase P's "Eighteen Essays in Non-Euclidean Geometry" [9], collectively discuss advancements in the domain of "Fundamentals of Geometry."

In the late nineteenth century, the investigation into the 'Fifth Axiom problem,' the 'Historical development of meta geometry,' 'Projective geometry,' 'Lobachevsky's geometric principles,' and 'Philosophical perspectives on meta geometry' in the writings of A. Bertrand and W. Russell formed part of the curriculum for the 'Fundamentals of Geometry' at Cambridge University." The research also looked into Kant's conception of 'metric geometries,' among other things [10].

Furthermore, while looking at the geometry courses given at the top 100 universities in developed countries, notably under the subject "Basics of Geometry," there is an emphasis on improving historical, philosophical, theological, and modern knowledge relating to geometry. Despite these attempts, it is worth noting that the existing curriculum in these schools does not thoroughly integrate modern geometric knowledge, showing a significant absence of its instruction in higher education.

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