PANJARADAGI IKKI ZARRACHALI DISKRET SCHRÖDINGER OPERATORINING XOS QIYMATLARI SONI VA UNING JOYLASHGAN O'RNI

Hamidov Sh.I . [1], Ibrogimov U.B. [2]
Sharof Rashidov nomidagi Samarqand davlat universiteti, Samarqand, O'zbekiston;
Sharof Rashidov nomidagi Samarqand davlat universiteti, Samarqand, O'zbekiston;

$\mathbb{T} = (-\pi, \pi]$ bir o'lchamli tor bo'lsin. $L^{2,e}(\mathbb{T}) \subset L^2(\mathbb{T})$ juft funksiyalar qism fazosi bo'lsin. Ikki zarrachali diskret Schrödinger operatorini $L^{2,e}(\mathbb{T})$ qism fazoda quyidagicha aniqlaymiz:

$$H_{\lambda\mu}(K) := H_0(K) + V_{\lambda\mu}, \quad K \in \mathbb{T}, \quad \lambda, \mu \in \mathbb{R}, \qquad (1)$$

bu yerda (qo'zg'almas) operator $H_0(K)$-$\varepsilon_K(p)$ funksiyaga ko'paytirish operatori

$$(H_0(K)f)(p) = \varepsilon_K(p)f(p), \qquad f \in L^{2,e}(\mathbb{T}).$$

Bunda

$$\varepsilon_K(p) = \varepsilon(p) + \gamma\varepsilon(K - p), \quad \gamma > 0.$$

Qo'zg'alish operatori $V_{\lambda\mu}$ quyidagi

$$[V_{\lambda\mu}f](p) = \frac{\lambda}{2\pi}\int_{\mathbb{T}} f(q)dq + \frac{\mu}{2\pi}\cos p\int_{\mathbb{T}} \cos q f(q)dq, \qquad f \in L^{2,e}(\mathbb{T}). \quad (2)$$

formula yordamida aniqlanadi.

Ravshanki, $V_{\lambda\mu}$-integral operator bo'lib, rangi ikkiga teng. Muhim spektr turg'unligi haqidagi Veyl teoremasiga ko'ra $H_{\lambda\mu}(K)$ operatorning $\sigma_{ess}(H_{\lambda\mu}(K))$ muhim spektri H_0 operatorning muhim spektri bilan ustma-ust tushadi, ya'ni,

$$\sigma_{ess}(H_{\lambda\mu}(K)) = \sigma(H_0(K)) = [\varepsilon_{min}(K), \varepsilon_{max}(K)]. \qquad (3)$$

Lemma 0.1 $z \in C \backslash [\varepsilon_{min}(0), \varepsilon_{max}(0)]$ soni $H_{\lambda\mu}(K)$ operatorning xos qiymati bo'lishi uchun

$$\Delta_{\lambda\mu}(z) = 0 \qquad (4)$$

bo'lishi zarur va yetarli.

Bu yerda

$$\Delta_{\lambda\mu}(z) = \begin{vmatrix} 1 + \lambda a(z) & \mu b(z) \\ \lambda b(z) & 1 + \mu c(z) \end{vmatrix}$$

va

$$a(z) = \frac{1}{2\pi}\int_{\mathbb{T}} \frac{dp}{\varepsilon_0(p) - z},$$

$$b(z) = \frac{1}{2\pi}\int_{\mathbb{T}} \frac{\cos p}{\varepsilon_0(p) - z} dp,$$

$$c(z) = \frac{1}{2\pi}\int_{\mathbb{T}} \frac{\cos^2 p}{\varepsilon_0(p) - z} dp \ .$$

Lemma 0.2 Yuqorida keltirilgan $a(\cdot)$, $b(\cdot)$ va $c(\cdot)$ funksiyalar uchun quyidagi tasdiqlar o'rinli:

- $\mathbb{C}\backslash[\varepsilon_{\min}(0), \varepsilon_{\max}(0)]$ da regulyar bo'ladi.

- $(\varepsilon_{\max}(0), +\infty)$ oraliqda manfiy va monoton o'suvchi bo'ladi.

- $(-\infty, \varepsilon_{\min}(0))$ oraliqda musbat va monoton o'suvchi bo'ladi.

Lemma 0.3 Berilgan $a(z)$, $b(z)$ va $c(z)$ funksiyalar uchun quyidagi assimptotik yoyilmalar o'rinli:

$$a(z) = \frac{1}{\sqrt{2+2\gamma}}(-z)^{-\frac{1}{2}} + o(-z)^{\frac{1}{2}}, \ \text{bu yerda} \ z \to \mathcal{E}_{\min}(0) - \quad (5)$$

$$b(z) = \frac{1}{\sqrt{2+2\gamma}}(-z)^{-\frac{1}{2}} - \frac{1}{1+\gamma} + o(-z)^{\frac{1}{2}}, \ \text{bu yerda} \ z \to \mathcal{E}_{\min}(0) - \quad (6)$$

$$c(z) = \frac{1}{\sqrt{2+2\gamma}}(-z)^{-\frac{1}{2}} - \frac{1}{1+\gamma} + o(-z)^{\frac{1}{2}}, \ \text{bu yerda} \ z \to \mathcal{E}_{\min}(0) - \quad (7)$$

va

$$a(z) = -\frac{1}{\sqrt{2+2\gamma}}(z-2-2\gamma)^{-\frac{1}{2}} + o(z-2-2\gamma)^{\frac{1}{2}}, \ \text{bu yerda,} \ z \to \mathcal{E}_{\max}(0) + \quad (8)$$

$$b(z) = -\frac{1}{\sqrt{2+2\gamma}}(z-2-2\gamma)^{-\frac{1}{2}} + \frac{1}{1+\gamma} + o(z-2-2\gamma)^{\frac{1}{2}}, \ \text{bu yerda,} \ z \to \mathcal{E}_{\max}(0) + \quad (9)$$

$$c(z) = -\frac{1}{\sqrt{2+2\gamma}}(z-2-2\gamma)^{-\frac{1}{2}} + \frac{1}{1+\gamma} + o(z-2-2\gamma)^{\frac{1}{2}}, \ \text{bu yerda,} \ z \to \mathcal{E}_{\max}(0) + \quad (10)$$

Lemma 0.4 Yuqorida olingan funksiyalarning asimptotikalarini hisobga olib har bir (λ, μ, γ) parametrlar uchun ushbu $\Delta^{e}_{\lambda\mu}(z)$ determinantning asimptotik yoyilmalari quyidagicha:

- $$\Delta^{e}_{\lambda\mu}(z) = S^{-}_{-\frac{1}{2}}(\lambda, \mu; \gamma)(-z)^{-\frac{1}{2}} + B^{-}_{0}(\lambda, \mu; \gamma) + o(-z)^{\frac{1}{2}}, \ \text{bu yerda} \ z \to \mathcal{E}_{\min}(0) -$$

bu yerda

$$S^{-}_{-\frac{1}{2}}(\lambda,\mu;\gamma)=\frac{\mu}{\sqrt{2+2\gamma}}+\frac{\lambda}{\sqrt{2+2\gamma}}+\frac{\lambda\mu}{(1+\gamma)\sqrt{2+2\gamma}},$$

$$B^{-}_{0}(\lambda,\mu;\gamma)=1-\frac{\mu}{1+\gamma}-\frac{\lambda\mu}{(1+\gamma)^2}.$$

•

$$\Delta^{e}_{\lambda\mu}(z)=S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)(z-2-2\gamma)^{-\frac{1}{2}}+B^{+}_{0}(\lambda,\mu;\gamma)+$$

$$+o(z-2-2\gamma)^{\frac{1}{2}},\qquad \text{bu yerda } z\to\mathcal{E}_{max}(0)+$$

bu yerda

$$S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)=-\frac{\mu}{\sqrt{2+2\gamma}}-\frac{\lambda}{\sqrt{2+2\gamma}}+\frac{\lambda\mu}{(1+\gamma)\sqrt{2+2\gamma}},$$

$$B^{+}_{0}(\lambda,\mu;\gamma)=1+\frac{\mu}{1+\gamma}-\frac{\lambda\mu}{(1+\gamma)^2}.$$

Asosiy teoremani shakillantirish uchun biz $S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)$ funksiya bilan bog'liq bo'lgan G^{+}_{2}, G^{+}_{1}, va G^{+}_{0} sohalarni , shuningdek $S^{-}_{-\frac{1}{2}}(\lambda,\mu;\gamma)$ funksiya bilan bog'liq bo'lgan G^{-}_{2}, G^{-}_{1}, va G^{-}_{0} sohalarni quyidagicha belgilab olamiz:

$$G^{+}_{2}=\{(\lambda,\mu)\in\mathbb{R}^2: S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)>0,\ \lambda>1+\gamma\}$$

$$G^{+}_{1}=\{(\lambda,\mu)\in\mathbb{R}^2:\ S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)=0,\ \lambda>1+\gamma\ \text{ yoki }\ S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)<0\}$$

$$G^{+}_{0}=\{(\lambda,\mu)\in\mathbb{R}^2:\ S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)=0,\ \lambda<1+\gamma\ \text{ yoki }\ S^{+}_{-\frac{1}{2}}(\lambda,\mu;\gamma)>0\}$$

va huddi shunday

$$G^{-}_{2}=\{(\lambda,\mu)\in\mathbb{R}^2: S^{-}_{-\frac{1}{2}}(\lambda,\mu;\gamma)>0,\ \lambda>-(1+\gamma)\}$$

$$G^{-}_{1}=\{(\lambda,\mu)\in\mathbb{R}^2:\ S^{-}_{-\frac{1}{2}}(\lambda,\mu;\gamma)=0,\ \lambda<-(1+\gamma)\ \text{ yoki }\ S^{-}_{-\frac{1}{2}}(\lambda,\mu;\gamma)<0\}$$

$$G^{-}_{0}=\{(\lambda,\mu)\in\mathbb{R}^2: S^{-}_{-\frac{1}{2}}(\lambda,\mu;\gamma)=0,\ \lambda>-(1+\gamma)\ \text{ yoki }\ S^{-}_{-\frac{1}{2}}(\lambda,\mu;\gamma)>0\}$$

Quyida $\lambda,\mu\in\mathbb{R}$ uchun $H^{e}_{\lambda 0}$ va $H^{e}_{0\mu}$ operatorlarning xos qiymatlari mavjudligi va ularning joylashgan o'rni uchun ba'zi natijalarni keltirib o'tamiz:

Theorem 0.5

• Ixtiyoriy $\lambda\neq 0$ uchun $H^{e}_{\lambda 0}$ operator yagona xos qiymatga ega:

agar $\lambda>0$ bo'lsa, $\zeta^{+}(\lambda)$ xos qiymat $(\varepsilon_{max}(0),+\infty)$ da yotadi, agar $\lambda < 0$ bo'lsa, $\zeta^-(\lambda)$ xos qiymat $(-\infty, \varepsilon_{max}(0))$ da yotadi.

- Ixtiyoriy $\mu \neq 0$ uchun $H^e_{0\mu}(0)$ operator yagona xos qiymatga ega:

agar $\mu > 0$ bo'lsa, $\zeta^+(\lambda)$ xos qiymat $(\varepsilon_{max}(0), +\infty)$ da yotadi,
agar $\mu < 0$ bo'lsa, $\zeta^-(\lambda)$ xos qiymat $(-\infty, \varepsilon_{max}(0))$ da yotadi.

Quyidagi belgilashlarni kiritamiz:

$$\zeta^+_{min} = \min\{\zeta^+(\lambda),\ \zeta^+(\mu)\},\ \ \zeta^+_{max} = \max\{\zeta^+(\lambda),\ \zeta^+(\mu)\}$$

$$\zeta^-_{min} = \min\{\zeta^-(\lambda),\ \zeta^-(\mu)\},\ \ \zeta^-_{max} = \max\{\zeta^-(\lambda),\ \zeta^-(\mu)\}$$

Quyida $\lambda, \mu \in \mathbb{R}$ uchun $H^e_{\lambda\mu}$ operatorning xos qiymatlari mavjudligi va ularning joylashgan o'rni uchun ba'zi natijalarni keltirib o'tamiz:

Theorem 0.6

- Faraz qilaylik, $(\lambda, \mu) \in G_{02} = G^-_0 \cap G^+_2$ bo'lsin, u holda $H^e_{\lambda\mu}(z)$ operator muhim spektrdan quyida xos qiymatga ega emas, muhim spektrdan yuqorida 2 ta xos qiymat mavjud va ular quyidagi shartlarni qanoatlantiriadi:

$$\varepsilon_{max}(0) < \zeta_1(\lambda, \mu) < \zeta_2(\lambda, \mu)$$

- Faraz qilaylik, $(\lambda, \mu) \in G_{01} = G^-_0 \cap G^+_1$ bo'lsin, u holda $H^e_{\lambda\mu}(z)$ operator muhim spektrdan quyida xos qiymatga ega emas, muhim spektrdan yuqorida 1 ta xos qiymat mavjud va quyidagi munosabat o'rinli:

$$\zeta(\lambda, \mu) > \varepsilon_{max}(0)$$

- Faraz qilaylik, $(\lambda, \mu) \in G_{11} = G^-_1 \cap G^+_1$ bo'lsin, u holda $H^e_{\lambda\mu}(z)$ operator muhim spektrdan quyida 1 ta xos qiymatga, muhim spektrdan yuqorida 1 ta xos qiymat mavjud va ular quyidagi shartlarni qanoatlantiriadi:

$$\zeta_1(\lambda, \mu) < \varepsilon_{min}(0) \quad \text{va} \quad \zeta_2(\lambda, \mu) > \varepsilon_{max}(0)$$

- Faraz qilaylik, $(\lambda, \mu) \in G_{10} = G^-_1 \cap G^+_0$ bo'lsin, u holda $H^e_{\lambda\mu}(z)$ operator muhim spektrdan yuqorida xos qiymatga ega emas, muhim spektrdan quyida 1 ta xos qiymat mavjud va quyidagi munosabat o'rinli:

$$\zeta(\lambda, \mu) < \varepsilon_{min}(0)$$

- Faraz qilaylik, $(\lambda, \mu) \in G_{20} = G^-_2 \cap G^+_0$ bo'lsin, u holda $H^e_{\lambda\mu}(z)$ operator muhim spektrdan yuqorida xos qiymatga ega emas, muhim spektrdan quyida 2 ta xos qiymat mavjud va ular quyidagi shartlarni qanoatlantiriadi:

$$\zeta_1(\lambda, \mu) < \zeta_2(\lambda, \mu) < \varepsilon_{max}(0)$$

ADABIYOTLAR:

1. I.N.Bozorov, Sh.I.Khamidov, S.N.Lakaev, " The number and location of eigenvalues of the two particle discrete Schrödinger operators," Lobachevskii J. Math. 43(11), 47–58, (2022).

2. S.N.Lakaev, Sh.I.Khamidov, " On the number and location of eigenvalues of the two particle Schrödinger operator on a lattice," Lobachevskii J. Math. 43 (12), 135–145, (2022).

3. S. N. Lakaev, I. U. Alladustova, " The exact number of eigenvalues of the discrete Schrцdinger operators in one-dimensional case," Lobachevskii J. Math. 42, 1294–1303 (2021).