GEOMETRIK MASALALARNI YECHISHDA CHEVA VA MENELAY TEOREMALARINI TADBIQI.

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"Ta'lim sifatini oshirish-Yangi O'zbekiston taraqqiyotining yakkayu yagona to'g'ri yo'lidir"-
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Annotatsiya: *Bugungi kunda Oliy ta'lim muassasalariga o'quvchilarni tayyorlashda bilim, ko'nikma va malakalarni salohiyatini yuksaltirish, yetarlicha tayyorgarlik talab qiladi. Buning uchun geometrik masalalarni yechishda Cheva va Menelay teoremalarini tadbiq etishga ko'proq e'tibor qaratishimiz, ishning samarali bo'lishida ahamiyatga egadir.*

Kalit so'zlar: *Tekislik, to'g'ri chiziqlarning bir nuqtada kesishishi, konkurentlik, uchburchak bissektrisasi, uchburchak balandliklari, uchburchak medianalari, uchburchak kesmalari, Cheva teoremasi, Menelay teoremasi*

Аннотация: *На сегодняшний день повышение потенциала знаний, умений и навыков при подготовке учащихся к высшим учебным заведениям требует достаточной подготовки. Для этого важно, чтобы мы уделяли больше внимания применению теорем Чева и Менелая при решении геометрических задач, чтобы работа была эффективной.*

Ключевые слова: *Плоскость, пересечение прямых в одной точке, конкуренция, биссектриса треугольника, высоты треугольника, медианы треугольника, пересечения треугольника, теорема Чева, теорема Менелая*

Annotation: *Today, Higher Education Institutions require sufficient training to increase the potential of knowledge, skills and abilities in the training of students. To do this, it is important that we pay more attention to the implementation of Cheva and Menelai theorems in solving geometric problems, that the work is effective.*

Keywords: *Plane, intersection of straight lines at one point, concurrent, triangle bisector, triangle Heights, triangle medians, triangle cuts, Cheva theorem, Menelai theorem*

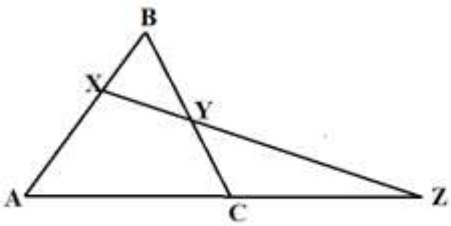
Xozirgi vaqtda yurtimiz uchinchi renesans davri ilm fan taraqqiyoti bosqichida.Shuning uchun bugungi kunda respublikamizda matematika faniga, ayniqsa matematik olimpiadalarga katta e'tibor berilmoqda. Xalqaro matematika olimpiadalarining tanlov savollarida (IMO Sharlist). Biz qarayotgan Cheva va Menelay teoremasi, uning analoglaridan juda qiziqarli masalalar berilgan. Biroq bugungi kunda mazkur teorema bo'yicha o'zbek tilidagi adabiyotlar juda kam. Shuning uchun maqola mavzusi bugungi kundagi dolzarb mavzulardan biri hisoblanadi.

Mazkur maqolaning maqsadi va vazifalari olimpiada bilan shug'ullanuvchi o'quvchilarga va ularning ustozlariga Cheva va Menelay teoremasi hamda undan kelib chiquvchi natijalar haqida ma'lumot berish hamda ushbu teoremaning qo'llanilishiga doir namunaviy misollar yechib ko'rsatishdan iboratdir.

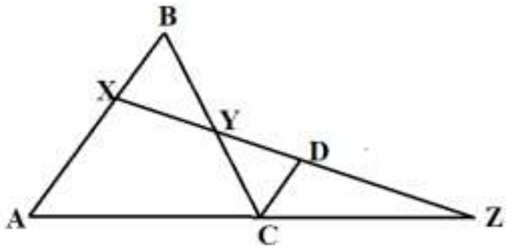
Faraz qilamiz, bizga ΔABC berilgan bo'lsin.

Menelay teoremasi. Agar ΔABC ning tomonlaridan yoki tomonlarini davomidan X,Y,Z nuqtalarni : X nuqta- AB da,Y- BC da, Z- CA da yotsa,u holda bu nuqtalar quyidagi shartni bajarsa ular bir to`gri chiziqda yotadi.

$$\frac{AX}{XB}\cdot\frac{BY}{YC}\cdot\frac{CZ}{ZA}=1$$



Δ Isbot: AB chiziqqa parallel bo'lgan C nuqtadan o'tuvchi CD chiziq o'tkazamiz.



Natijada quyidagicha o'xshash uchburchaklar xosil bo'ladi

AXIICD ekanligidan =>ΔAXZ~ΔCDZ; bu yerda ∠XAZ=∠DCZ; ∠AXZ=∠CDZ;

∠ BXY=180⁰- ∠ AXD=180⁰- ∠ CDZ= ∠ CDY; vertikal burchaklar ∠ BYX=CYD; ΔBXY~ΔCDY

Demak, o'xshashlik alomatiga ko'ra $\frac{AX}{DC}=\frac{AZ}{CZ}$; => $DC=\frac{AX\cdot CZ}{AZ}$; $\frac{BY}{YC}=\frac{XB}{DC}$; =>

$$DC=\frac{XB\cdot YC}{BY}$$

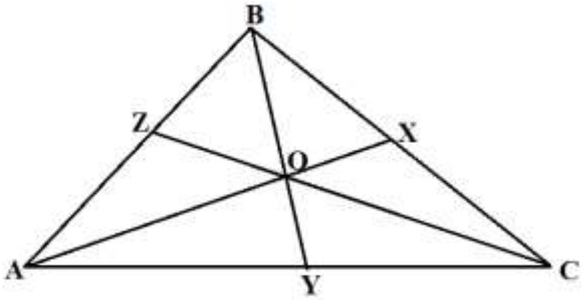
Xosil qilingan ikki tenglikni hadma-had bo'lamiz: $\frac{DC}{DC}=\frac{\frac{AX\cdot CZ}{AZ}}{\frac{XB\cdot YC}{BY}}$; => $1=\frac{AX\cdot CZ\cdot BY}{XB\cdot YC\cdot AZ}$; =>

$\frac{AX}{XB}\cdot\frac{BY}{YC}\cdot\frac{CZ}{ZA}=1$ teorema isbotlandi. ▲

Faraz qilamiz, bizga ΔABC berilgan bo'lsin.

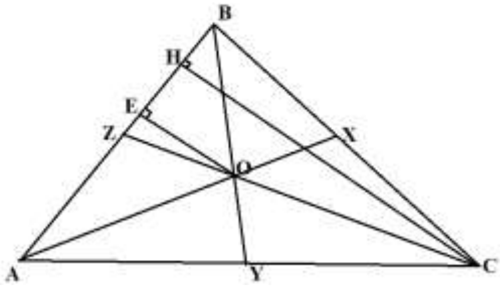
Ta'rif. Uchburchakning biror uchi bilan qarama-qarshi tomonidagi biror nuqtani tutashtiruvchi kesma cheviana deyiladi. Demak, uchburchakda X,Y,Z nuqtalar mos ravishda BC, CA, AB tomonlarga tegishli bo'lsa, u holda kesmalar chevianalar bo'ladi.

Teorema.(Cheva) Agar ABC uchburchakda AX, BY va CZ chevianalar konkurent bo'lsa, u holda



$\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}\cdot\frac{|AZ|}{|ZB|}=1$ munosabat oʻrinli boʻladi.

Δ Isbot1: Faraz qilaylik AX, BY, CZ kesmalar O nuqtada kesishsin. ΔAZC va ΔBZC lar umumiy CH balandlikka ega.



Shuning uchun $\frac{S_{AZC}}{S_{BZC}}=\frac{\frac{1}{2}\cdot|AZ|\cdot|CH|}{\frac{1}{2}\cdot|BZ|\cdot|CH|}=\frac{|AZ|}{|BZ|}$

Xuddi shunday ΔAZO va ΔBZO lar umumiy OE balandlikka ega.

Demak, $\frac{S_{AZO}}{S_{BZO}}=\frac{\frac{1}{2}\cdot|AZ|\cdot|OE|}{\frac{1}{2}\cdot|BZ|\cdot|OE|}=\frac{|AZ|}{|BZ|}$ Demak, bu yerdan $\frac{|AZ|}{|BZ|}=\frac{S_{AZC}}{S_{BZC}}=\frac{S_{AZO}}{S_{BZO}}$ munosabatga ega bo'lamiz. Biz proporsiyaning quyidagi xossasidan foydalanamiz:

Lemma. $\frac{a}{b}=\frac{c}{d}=\frac{a-c}{b-d}$ (b≠d)

Isbot. $\frac{a}{b}=\frac{c}{d}=\gamma$; bo'lsa, $\frac{a-c}{b-d}=\gamma$ ekanligini isbotlaymiz. $\frac{a}{b}=\frac{c}{d}=\gamma$dan a=γb; c=γd

$\frac{a-c}{b-d}=\frac{\gamma b-\gamma d}{b-d}=\gamma$ isbotlandi

Bu Lemmani $\frac{|AZ|}{|BZ|}=\frac{S_{AZC}}{S_{BZC}}=\frac{S_{AZO}}{S_{BZO}}$ munosabat uchun qo'llasak,

$$\frac{|AZ|}{|BZ|}=\frac{S_{AZC}}{S_{BZC}}=\frac{S_{AZO}}{S_{BZO}}=\frac{S_{AZC}-S_{AZO}}{S_{BZC}-S_{BZO}}=\frac{S_{AOC}}{S_{BOC}}$$

Tenglikka ega bo'lamiz.

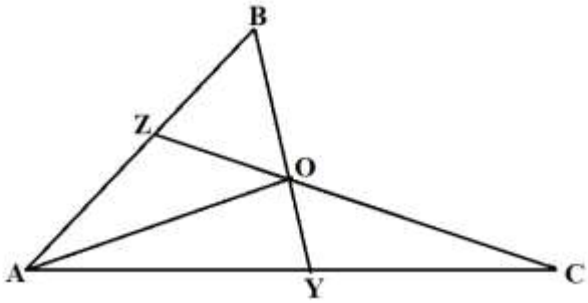
Aynan shu yo'l bilan $\frac{|BX|}{|XC|}=\frac{S_{AOB}}{S_{AOC}}$ va $\frac{|CY|}{|YA|}=\frac{S_{BOC}}{S_{AOB}}$ ekanligi ko'rsatiladi.

U holda $\frac{|BX|}{|XC|}\cdot\frac{|CY|}{|YA|}\cdot\frac{|AZ|}{|ZB|}=\frac{S_{AOB}}{S_{AOC}}\cdot\frac{S_{BOC}}{S_{AOB}}\cdot\frac{S_{AOC}}{S_{BOC}}=1$ tenglik kelib chiqadi.

Shuni isbotlash kerak edi. ▲

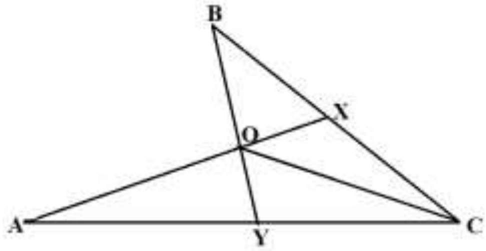
Δ Isbot2:(Menelay teoremasi yordamida)

Chizmada berilgan uchburchakning quyidagi bo'lagini olamiz va Menelay teoremasini qo'llaymiz



$\frac{CY}{AC}\cdot\frac{AZ}{ZB}\cdot\frac{BO}{OY}=1;$

Endi boshqa bo'lagi uchun Menelay teoremasini qo'llaymiz



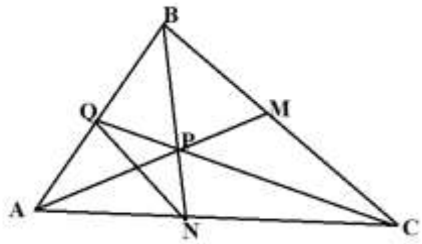
$\frac{AY}{AC}\cdot\frac{CX}{BX}\cdot\frac{BO}{OY}=1$; yuqorida xosil bo'lgan ikki nisbatni mos bo'laklarini bo'lamiz

$\frac{\frac{CY}{AC}\cdot\frac{AZ}{ZB}\cdot\frac{BO}{OY}}{\frac{AY}{AC}\cdot\frac{CX}{BX}\cdot\frac{BO}{OY}}=\frac{1}{1}$; o'xshash hadlarni qisqartiramiz $\frac{CY}{AY}\cdot\frac{AZ}{ZB}\cdot\frac{BX}{CX}=1$ tenglikka ega bo'lamiz.

Teorema isbotlandi. ▲

Cheva va Menelay teoremasining qo'llanishiga doir matematik masalalar

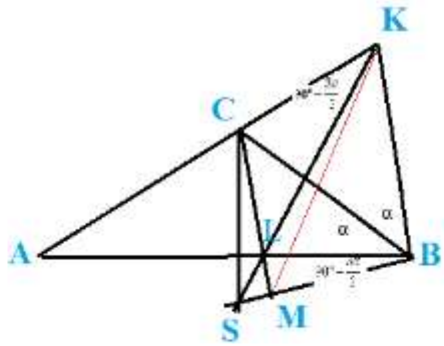
1-masala.(Mathematical Reflections, №2, 2007) ABC uchburchakning MA medianasi bilan NB bissektrisasi P nuqtada kesishadi. CP va AB to'g'ri chiziqlar Q nuqtada kesishadi. BNQ uchburchak teng yonli ekanligini isbotlang.



Δ Isbot: MA, NB va CQ chevianalar uchun Cheva teoremasini qo'llaymiz:

$\frac{AQ}{BQ}\cdot\frac{BM}{MC}\cdot\frac{CN}{NA}=1$ MA mediana ekanligidan BM=MC tenglik o'rinli demak, $\frac{BM}{MC}=1$ U holda (1) ga ko'ra $\frac{AQ}{BQ}\cdot\frac{CN}{NA}=1$ bundan $\frac{AQ}{BQ}\cdot\frac{NA}{CN}$ va natijada Fales teoremasiga ko'ra QN II BC BN kesma QN va BC ni kesayotgani uchun => ∠NBC=∠BNQ. BN-bissektrisa. => ∠QBN=∠NBC=> ∠QBN=∠QNB=∠NBC ya'ni ∠QNB=∠QBN u holda teng yonli. Uchburchak bo'ladi. Shuni isbotlash kerak edi.▲

2-masala. (Silk Road MO 2011).Teng yonli ABC uchburchakda ∠ACB>90^0 AC tomonni C nuqtadan davom ettirib, shu davomda ∠KBC=∠ABC tenglikni qanoatlantiradigan K nuqta olingan. ∠BKC va ∠ACB burchaklar bissektrisalari S nuqtada, AB va KS to'g'ri chiziqlar L nuqtada, BS va CL to'g'ri chiziqlar esa M nuqtada kesishsin. KM to'g'ri chiziq BC tomonning o'rtasidan o'tishini isbotlang.



Δ Isbot: KM to'g'ri chiziq BC ning o'rtasidan o'tishini isbotlash uchun

$\frac{BK}{CK} = \frac{\sin\angle MKC}{\sin\angle MKB}$ ni isbotlasak yetarli.CAB=$\angle$ABC=α bo'lsin. => $\angle$KBC=α =>$\angle$AKB=π-3α =>

$\angle$AKL=$\angle$LKB=90^0- $\frac{3\alpha}{2}$ Bizga ma'lumki uchburchakning bitta ichki va ikkita tashqi bissektrisalari bir nuqtada kesishadi. ΔBKC da SB kesma tashqi bissektrisadir.

=> $\angle$CBS=90^0- $\frac{\alpha}{2}$ => $\angle$LBS=90^0- $\frac{3\alpha}{2}$ => $\angle$AKS=$\angle$ABS=> AKBS to'rtburchakkka tashqi aylana chizish mumkin. ΔBKC da M nuqta uchun Cheva teoremasini qo'llaymiz:

$$\frac{\sin\angle BCM}{\sin\angle MCK}\cdot\frac{\sin\angle MKC}{\sin\angle MKB}\cdot\frac{\sin\angle MBK}{\sin\angle MBC} = 1 \Rightarrow \frac{\sin\angle MKC}{\sin\angle MKB} = \frac{\sin\angle MKC}{\sin\angle BCM}\cdot\frac{\sin\angle MBC}{\sin\angle MBK} \quad (1)$$

$$\frac{\sin\angle MCK}{\sin\angle BCM} = \frac{\sin(\pi-\angle ACL)}{\sin\angle BCL} = \frac{\sin\angle ACL}{\sin\angle BCL} = \frac{AL}{BL} = \frac{AK}{BK} = \frac{\sin2\alpha}{\sin\alpha} => \frac{\sin\angle MCK}{\sin\angle BCM} = \frac{\sin2\alpha}{\sin\alpha} \quad (2)$$

$$\frac{\sin\angle MBC}{\sin\angle MBK} = \frac{\sin(90^0-\frac{3\alpha}{2}+\alpha)}{\sin(90^0-\frac{3\alpha}{2}+2\alpha)} = \frac{\sin(90^0-\frac{\alpha}{2})}{\sin(90^0+\frac{\alpha}{2})} = \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} =>$$

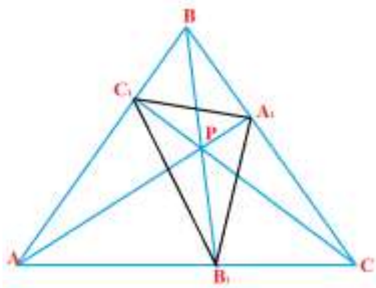
$\frac{\sin\angle MBC}{\sin\angle MBK} = 1$ (3) (2) va (3) larni (1) ga qo'yamiz:

$$\frac{\sin\angle MKC}{\sin\angle MKB} = \frac{\sin\angle MCK}{\sin\angle BCM}\cdot\frac{\sin\angle MBC}{\sin\angle MBK} = \frac{\sin2\alpha}{\sin\alpha} => \frac{\sin\angle MKC}{\sin\angle MKB} = \frac{\sin2\alpha}{\sin\alpha} \quad (4)$$

Bizga ma'lumki, $\frac{BK}{KC} = \frac{\sin2\alpha}{\sin\alpha}$ (5)

(4) va (5) ga ko'ra $\frac{BK}{KC} = \frac{\sin\angle MKC}{\sin\angle MKB} => KM$ to'g'ri chiziq BC kesmaning o'rtasidan o'tadi. => isbotlandi.▲

3-masala. (IMO Shortlist 1996). ABC muntazam uchburchakning ichida ixtiyoriy P nuqta olingan. AP, BP, CP to'g'ri chiziqlar BC, CA, AB kesmalarni mos ravishda, A_1, B_1, C_1 nuqtalarda kesadi. Isbotlang: $A_1B_1\cdot B_1C_1\cdot C_1A_1 > A_1B\cdot B_1C\cdot C_1A$



Δ Isbot: Kosinuslar teoremasidan foydalanamiz:

$A_1B_1 = \sqrt{B_1C^2 + A_1C^2 - 2B_1C \cdot A_1C \cdot \cos 60^0} = \sqrt{B_1C^2 + A_1C^2 - 2B_1C \cdot A_1C \cdot \frac{1}{2}} = \sqrt{B_1C^2 + A_1C^2 - B_1C \cdot A_1C} \Rightarrow A_1B_1 = \sqrt{B_1C^2 + A_1C^2 - B_1C \cdot A_1C}$

Bizga ma'lumki, $(B_1C - A_1C)^2 \geq 0 => B_1C^2 - 2B_1C \cdot A_1C + A_1C^2 \geq 0$

⇨ $B_1C^2 - 2B_1C \cdot A_1C + A_1C^2 \geq B_1C \cdot A_1C$

⇨ $A_1B_1 = \sqrt{B_1C^2 - B_1C \cdot A_1C + A_1C^2} \geq \sqrt{B_1C \cdot A_1C}$

⇨ $A_1B_1 \geq \sqrt{B_1C \cdot A_1C}$ (1) Xuddi shunday yo'l bilan quyidagi tengsizliklarni hosil qilamiz: $B_1C_1 \geq \sqrt{AC_1 \cdot AB_1}$ (2) $C_1A_1 \geq \sqrt{BC_1 \cdot A_1B}$

⇨ (1), (2), (3) tengsizliklarni ko'paytirib yuborsak

⇨ $A_1B_1 \cdot B_1C_1 \cdot C_1A_1 \geq \sqrt{B_1C \cdot A_1C \cdot AC_1 \cdot AB_1 \cdot BC_1 \cdot A_1B}$ (4) ga ega bo'lamiz.

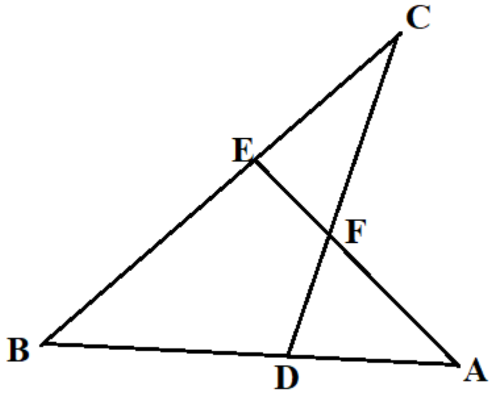
Cheva teoremasiga ko'ra $A_1B \cdot BC_1 \cdot AC_1 \cdot A_1B = AB_1 \cdot BC_1 \cdot A_1C$

⇨ (4) ga ko'ra : $A_1B_1 \cdot B_1C_1 \cdot C_1A_1 \geq \sqrt{B_1C \cdot A_1C \cdot AC_1 \cdot AB_1 \cdot BC_1 \cdot A_1B} = \sqrt{(B_1C \cdot A_1B \cdot AC_1)^2} == B_1C \cdot A_1B \cdot AC_1 =>$

⇨ $A_1B_1 \cdot B_1C_1 \cdot C_1A_1 => B_1C \cdot A_1B \cdot AC_1$ shuni isbotlash kerak edi. ▲

Cheva va Menelay teoremasining qo'llanishiga doir fizik masalalar

1.Moddiy nuqta A nuqtadan chiqib, rasmda berilgan AF, FD, BD, BC, CD chiziq bo'yicha xarakat qildi va D nuqtada to'xtadi.Agar BE=EC, AF=3, BD=4, BC=10, CD=4, FD=1 ga teng bo'lsa, moddiy nuqtani ko'chishini va bosib o'tgan yo'lini toping.



Berilgan

BE=EC, AF=3, BD=4, BC=10, CD=4, FD=1

Topish kerak: AD=?, AF+FD+BD+BC+CD=?

Δ Yechish: Menelay teoremasiga ko'ra

$\frac{AD}{AB} \cdot \frac{BE}{EC} \cdot \frac{CF}{FD} = 1$ bo'lsa, u holda

BC=10, BE=EC=› BE=EC=5, BD=4, AF=3, CD=4, FD=1=›CF=3, AD=x, AB=AD+BD=x+4 ekanligi kelib chiqadi

Topilgan qiymatlarni o'rniga olib borib qo'yamiz:

$\frac{x}{x+4} \cdot \frac{5}{5} \cdot \frac{3}{1} = 1$; bu yerdan 3x=x+4; 2x=4; x=2 ekanligini topamiz:

Demak, Javob: moddiy nuqtani ko'chishi AD=2 va bosib o'tgan yo'li

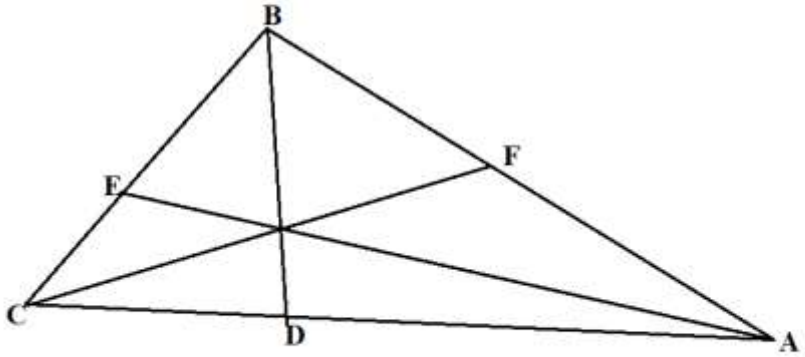
AF+FD+BD+BC+CD= 3+1+4+10+4=22 ga teng. ▲

2.Moddiy nuqta rasmdagi kabi A nuqtadan chiqib, B nuqta tomon xarakatmoqda va B nuqtaga yetgach o'z yo'nalishini C nuqtaga va C nuqtadan D nuqtaga o'zgartirdi.D nuqtada moddiy nuqta xarakatdan to'xtadi. Agar AF=FB; AB=16; BE=3; EC=2; CD=4 ga teng bo'lsa, moddiy nuqtani bosib o'tgan yo'li va ko'chishini toping.

Berilgan

AF=FB; AB=16; BE=3; EC=2; CD=4

Topish kerak: AB+BC+CD=? AD=?



Δ Yechish: AB+BC+CD= AB+BE+EC+CD=16+3+2+4=25

⇨ AD=x, deb olsak, va AF=FB; AB=16=> AF=FB=8 Cheva teoremasiga ko'ra $\frac{AF}{FB}\cdot\frac{BE}{EC}\cdot\frac{CD}{AD}=1$; $\frac{8}{8}\cdot\frac{3}{2}\cdot\frac{4}{x}=1$; $\frac{6}{x}=1$; x=6

XULOSA

Mazkur maqola Cheva va Menelay teoremasi, bu teoremalarni isboti, ularni geometrik va fizik masalalar yechishga tadbiqini "Temurbeklar maktabi", barcha akademik litsey va maktab o'quvchilarini 10, 11-sinf o'quvchilariga tushuntirib berishga qaratilgan. Maqolada uchburchaklar kesmalarining konkurent bo'lishi uchun Cheva hamda Menelay teoremasidan kelib chiqadigan muhim natijalar isbotlab ko'rsatilgan, hamda teorema yordamida yechiladigan bir nechta olimpiada masalalari, dars jarayonida uchraydigan murakkab masalalar berilgan. Maqolada olingan natijalar va usullar geometriya va fizikada muhim tushunchalardan biri bo'lgan uchburchak kesmalarining konkurent bo'lishi masalasi hal qilishda qo'llaniladi. Shuningdek ushbu maqoladan olimpiadalarga tayyorgarlik ko'rayotgan maktab va litsey o'quvchilari ham foydalanishi mumkin.

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